

# Solid Geometry:

OR,

Foundation of Measuring,

Of all Manner of

# Solid Bodies.

Most accurately Demonstrated in various Propositions; and applied to Practice in the exact Mensuration of *Timber* or *Stone* (though never so irregular) being of great Use to all *Carpenters, Bricklayers* and *Masons, &c.*

BUT

Particularly intended, for the true Gauging of all Manner of Vessels, of what Form or Figure soever; either in whole or in part: As Wine, Beer-Casks, Brewers-Tuns and Coppers, Backs or Coulers. Being very useful for all *Gaugers, Generals, Surveyors, Supervisors*, and all other *Gaugers, &c.*

*To which is now added,*

A very necessary Appendix explaining and facilitating the former Work, in the Practical Part thereof; being of singular Use to all Persons concern'd therein: The like not Extant in the English Tongue.

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*Recommended to the Reader by the Royal Society.*

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LONDON: Printed for J. Conyers at the Bible and Anchor near St. Peter's Church in Cornhill, 1703.

*Solid Bodies*



Foundation of Mechanics  
Of all Manner of

# Solid Bodies.

Most accurately described in various Pro-  
positions; and applied to Practice in the ex-  
act Measurement of Vessels or Solids (though  
never to interest) being of great Use to all  
Engineers, Architects and Astronomers.

BUT

Particularly necessary for the true Gauging of  
all Vessels of what Form or Figure so-  
ever; and in which the Author has given the  
Rules for finding the true Contents of Casks,  
Galls, Brewsters, and Coppers, &c. &c. &c.  
And very useful for all Gaugers, &c. &c. &c.

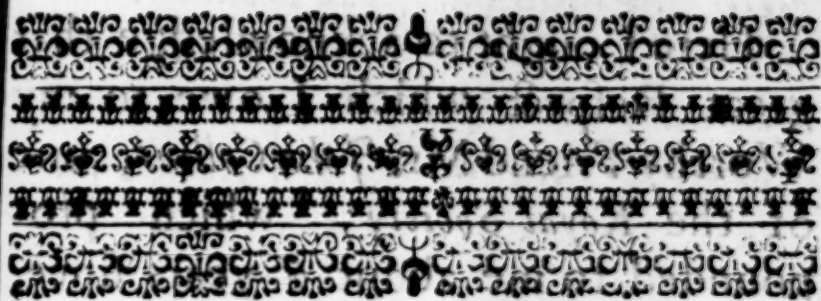
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Printed by W. Rogers at the Bible and Anchor  
near St. Pauls Church in Cornhill, 1753.





# To the R E A D E R.



Bout seven years since  
I resolved these fol-  
lowing propositions,  
with others of the  
like nature ; but did  
not intend ever to  
have published them : But being over-  
perswaded by some of my Friends,  
I have here made so many of them  
publick, as I thought would be con-  
venient to what here is intended.

## *To the Reader.*

Here I ought to acknowledge that great Respect which I owe to my Worthy Friend Mr. *John Collins*, one of the *Royal Society*; which propounded to me about 6 years since, that proposition of the second Sections of the *Sphere* and *Spheroide*; and also in giving me Information, and helping me to such Books as might be most advantageous: Further, not only I, but all Lovers of the Mathematicks are very much obliged to this our Worthy Friend for his good Intelligence, great care in sending for Books from beyond the Seas, and his continual love in promoting the Mathematicks, and Mathematical Men.

For

## To the Reader.

Forasmuch as throughout this little Tract, here is much use made of *Parallelepipedons*, *Prismes* and *Pyramides*: I think fit, a little to insist thereon; chiefly for the young Learners sakes, for which this is only intended.

In the *Diagram*,

Let there be a *Solid*, as  $POHG$   $VFIR$ ; the *plane*  $POHG$  parallel equal and alike to the *plane*  $RI FV$ ; Further,  $POIR$  is parallel equal and like to the *plane*  $GH FV$ : Yet further, the *plane*  $OHFI$  is parallel equal and like the *plane*  $PRVG$ : such a *Solid* is called a *Parallelepipedon*.

Let there be a *Solid* as  $GHDEV$   $F$ , the *plane*  $GEV$  is parallel equal and like the *plane*  $HDF$ ; Further, the *planes*  $HGFV$  and  $DEVF$  are alike and the line  $FV$  common to both: such a *Solid* is called a *Prisme*.

Let there be a *Solid* as  $HBCDF$ , the *planes*  $BCF$ ,  $CDF$ ,  $DHF$  and

## To the Reader.

H B F all meet at the point F, such a *Solid* is called a *Pyramide*.

Further,

*Pyramides*, *Prismes* and *Parallelepipeds* upon the same *Base* and *Altitude*, are as, one, one and a half, and three, that is, if the *Pyramide* be 4, the *prisme* will be 6, and the *parallelepipedon* will be 12.

*An Example in Numbers.*

If P G, 6; P O, 8; and H F the *Altitude* be 9; Then multiply 6 by 8, and the product will be 48, the *Area* P O H G, this *Area* multiplyed by 9 the *Altitude*, the product is 432, the *parallelepipedon* P O H G V F I R, if the *Area* 48 be multiplyed by half 9 that is  $4\frac{1}{2}$  the product will be 216, equal to a *prisme* of the *Base* P O H G and altitude H F.

If the *Area* P O H G, 48, be multiplyed by one third of 9, that is by 3, the product will be 144 equal to a *pyramide* whose *Base* is P O H G and altitude H F.

To

## To the Reader.

To find the solidity of the prisme  $G H D E V F$ .

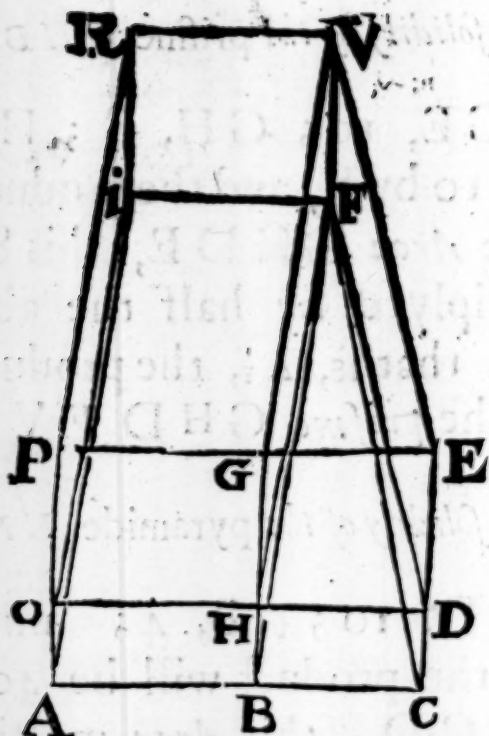
Let  $GE$ , 10;  $GH$ , 8;  $HF$  9;  
multiply 10 by 8, and the product will  
be 80 the *Area*  $G H D E$ , this 80 be-  
ing multiplied by half the altitude  
 $HF$ , 9; that is, 4½, the product will  
be 360, the *prisme*  $G H D E V F$ .

To find the solidity of the pyramide  $H B C D F$ .

Let  $HD$ , 10;  $HB$ , 4; multiply  
10 by 4 the product will be 40, the  
*Area*  $H B C D$ , this *Area* multiplied  
by 3, that is one third of the Altitude  
 $HF$ , the product will be 120, the py-  
ramide  $H B C D F$ .



## To the Reader.



*To find the solidity of a Solid, that hath one of its Bases an Ellipsis and the other Base a Circle, these two Bases are supposed to be parallel.*

In such a *Solid*, the sides being continued will never meet at a point as in circular and *Elliptick* cones, and therefore the general Rule will not resolve these kind of *Solids* : therefore consider it thus ;

Let

## To the Reader.

Let  $PE$  be equal to the Transverse diameter in an *Ellipsis*,  $PA$  its Conjugate diameter. Let  $RV$  and  $RI$  be the diameters of a Circle. First, Let there be made a Rectangled figure of the Transverse and Conjugate diameters of the *Ellipsis*, as  $PACE$ . Let there be a square made of the diameter  $RV$ , as  $RIFV$ . Let the altitude of the *Solid* be  $HF$ , find the solidity of the *Solid*  $PACEVFIR$ , thus, Let it be cut into as many *parallelepipeds*, *prismes*, & *pyramides* as are necessary: then find the solidity of those *parallelepipeds*, *prismes* and *pyramides*, as before is taught, those *parallelepipeds*, *prismes* & *pyramides* being added together will be equal to the whole *Solid*, thus, the *parallelepipedon*  $POHG VFIR$ , more the *prisme*  $GEDHFV$ , more the *pyramide*  $DCBHF$ , more the *prisme*  $HBAOIF$ ; these 4 *Solids* being added together make the whole *Solid*  $ACEPRIFV$ . For the whole  
is

## To the Reader.

is equal to all its parts taken together.

*Axiom. 19. 1. of Euclid.*

The *Areas* of Circles, are in proportion one to another, as the Squares of their Diameters 2. 12. of *Euclid.*

The *Areas* of *Ellipsis* are in proportion one to another, as the Rectangled figures of their Transverse and Conjugate Diameters. *Archimedes* the 7. Prop. of *Conoides* and *Spheroides.*

The *Area* of a Circle, is to the *Area* of an *Ellipsis*; as the Square of the Diameter of that Circle, is to the Rectangled figure of the Transverse and Conjugate Diameters of that *Ellipsis*, the 6th. of *Archimedes Conoides* and *Spheroides.*

As 14, is to 11; so is the square of the Diameter of any Circle to the *Area* of that Circle. prop. the 2. of *Archimedes Circul. Dimens.*

The *Solid* PACEVFIR may be supposed to have the nature of infinite Rectangled figures, parallel one to another,

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another, and parallel to the parallel planes  $ACEP$  and  $IFVR$ , and from the *parallelogram*  $ACEP$  to the square  $RIFV$ , every *parallelogram* coming nearer and nearer a square, even until they come to the square  $RIEV$  itself, so in *Elliptick Solids* adscribed in such *plane Solids*; the *Areas* of *Ellipses* comes nearer and nearer to the *Area* of the Circle whose Diameter is  $RV$ . Therefore, as 14, is to 11, so are such *plane solids* to such *elliptick solids*, adscribed in those *plane solids*.

### An Example in Numbers.

Let  $RV$  or  $RI$  be equal to 3;  $PE$ , 6;  $PA$ , 4;  $HF$ , 9; Therefore  $GE$ , 3;  $OA$ , 1; Therefore  $POHG$ , 9; equal  $RIFV$ , which being multiplied by 9, the altitude  $HF$ ; makes 81, equal to the *parallelepipedon*  $POHGVFIR$ .  $GE$ , 3; being multiplied by  $GH$ , 3; the product is  $GHDE$ , 9; which being multiplied by half the altitude  $HF$ , that is,

## To the Reader.

is, 4; the product is 40; equal to the *prisme* GHDEVF. HD, 3; HB, 1; therefore HBCD is equal to 3; which being multiplied by one third of the altitude HF, that is, 3; the *pyramide* HBCDF will be equal to 9; BA, 3; AO, 1; therefore ABHO will be equal to 3, which being multiplied by half HF the altitude, that is, by 4; the product is 12; equal to the *prisme* OABHFI. Now these 4 composed Numbers being added together make 144 equal to the whole *frustum pyramide* PACEVFIR. Then as  $14 : 11 :: 144 : 113 \frac{1}{3}$  the irregular *Elliptick solid*, whose Transverse PE, and conjugate PA diameters at the *Elliptick Base* are 6 and 4, and the diameters of the *Circular Base* is RV 3.

Here note,

Although *Archimedes* gives the proportion of the square of the diameter of any Circle, to the *area* of that Circle;



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Circle; as 14, is to 11. And also; that the diameter of any Circle is to the circumference of that Circle, as 7, is to 22; Yet you are not to understand that these proportions are in their just values; But that they are the least numbers that will best agree to that approach.

If those numbers be thought not to agree near enough to truth, instead of those, these may be used; for, 14 and 11, take 452 and 355; for 7 and 22, take 113 and 355.

### Further note.

In the 24th. proposition there is somewhat said of *Cylindrick hoofs*. Those *cylinders* ought to be upright and not inclining *cylinders*.

### Yet further note:

By reason of much business, I could not attend the *Press* as I ought to have done; and by that means there are considerable faults, which I desire may be corrected before the Book is read, other-

## *To the Reader.*

otherwise there may rise a misapprehension of what there is intended.

Lastly note,

Almost at the beginning of the first proposition you may find it printed, Thus,  $CN: NZ :: EH: HZ$ , 4: 6. the meaning is thus, as  $CN$  is to  $NZ$ ; so is  $EH$ , to  $HZ$ , by the fourth prop. of the sixth of *Euclid*; The like in the rest.



*R. A.*



## PROPOSITION I.

To find the solidity of a Frustum Pyramide,  
whose bases are parallel and a like.

**L**Et ABCNHEDF be a  
frustum pyramide, whose soli-  
dity is required; Let the Lines  
AF, BD, CE, and NH be  
continued till they meet at Z;  
Let GO, be the height of the *frustum pyra-  
mid*, OZ the height of the Continuation;  
Let FDEH beset at the Angle N, as NR  
QK. Then  $CN : NZ :: EH : HZ$ , 4. 6.

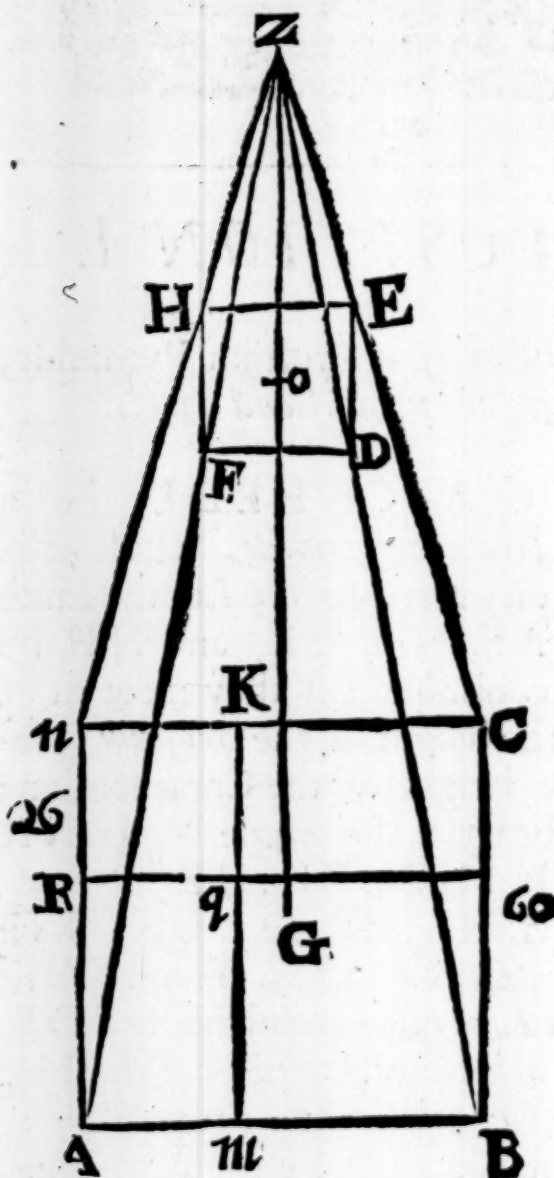
And  $CN : EH :: NZ : HZ$ , 16. 5.  $CN - EH :$   
 $EH :: NZ - HZ : HZ$ , 17. 5. That is,  $CK :$   
 $KN :: NH : HZ$ , because the planes HFDE,  
and NABC are parallel, they cut the Line  
ZG, in G and O in the same proportion as  
they cut NZ, in N and Z, by 17. 11. Thus,  
 $NH : HZ :: GO : OZ$ , Therefore  $CK : KN ::$   
 $GO : OZ$ , 11. 5. OZ being found, which  
B added

( 2 )

added to G O makes the whole Altitude G Z;  
by which find the whole *pyramide* N A B C Z,

which done,  
find the *pyra-  
mide* H F D  
E Z, by the  
7. 12. then N  
A B C Z less  
H F D E Z, the  
Remainder is  
the *frustum*  
H F D E C B  
A N.

A 2<sup>d</sup> way  
may be thus;  
First to prove  
that A K is a  
mean propor-  
tion between  
A C and F  
E; then C  
N: N K:: C  
A: A K, 1. 6.  
A N: N R::  
N M: N Q,  
1. 6. C N:  
N K:: A N:  
N R, 7. 5.  
C A: A K::



N M: N Q, 11. 5.

Further,



Further, if  $GO$  the Altitude of the *frustum*  $FDEHCBAN$ , be multiplied by  $AC$  more  $AK$  more  $KR$ , that Product will be the triple of the said *frustum*; for the parallelepipedon made of  $GZ$  the altitude of the *pyramide* in the Base  $NABC$  is triple of the *pyramide*  $NABCZ$ , by the same reason the parallelepipedon made of  $GO$  in  $AC$ , together with the parallelepipedon made by  $ZO$  in  $AC$  is triple of the said *pyramide* by 7. 12. if the parallelepipedon made of  $OZ$  in  $FE$ , that is, in  $RK$ , that is, the triple of the *pyramide*  $HFEZ$  be taken from the parallelepipedon  $NABCZ$ , there remains that which is made of  $GO$  in  $AC$ , together with those that are made of  $OZ$  in  $AQ$  and the same  $OZ$  in  $MC$ , the triple of the *frustum*  $ABCNHEDF$ , by 5. 5. for,  $CK:KN::GO:OZ$ ,  $CK:KN::AR:RN$ , Therefore  $GO:OZ::AR:RN$ , and  $CK:KN::CM:MN$ , 4. 6.  $AR:RN::AQ:QN$ , therefore  $GO:OZ::CM:MN$ , 11. 5.  $GO:OZ::AQ:QN$ , therefore  $GO$  in  $MN=OZ$  in  $CM$ ,  $GO$  in  $QN=OZ$  in  $AQ$ , add  $GO$  in  $AC$  to both, that is,  $GO$  in  $MN$ , more  $GO$  in  $QN$ , more  $GO$  in  $AC$ , is equal to  $OZ$  in  $CM$ , more  $OZ$  in  $AQ$ , more  $GO$  in  $AC$ ; but  $OZ$  in  $CM$ , more  $OZ$  in  $AQ$ , more  $GO$  in  $AC$  are triple of the said *frustum*; therefore  $GO$  in  $MN$ , more  $GO$  in  $QN$ ,



more GO in AC are equal to the triple of the said *frustum*.

*An Example in Numbers, and first of the first.*

CK, 34 : KN, 26 :: GO, 68 : OZ, 52 ; then 68 more 52 = 120, GZ : then AC 3600 ; in GZ, 120 ; is equal to 432000, a third part is 144000, ABCNZ, FE, 676 in OZ, 52 is equal to 35152.

A third part is  $11717\frac{1}{3}$ , HFDEZ, 144000 less  $11717\frac{1}{3}$  is equal to  $132282\frac{2}{3}$  the solidity of the *frustum* ACEF.

*The Second way.*

AC, 3600 more FE, 676, more AK, 1560 the sum of those are 5836, which multiplied by GO, 68, is 396848, a third part is  $132282\frac{2}{3}$ . These ways *Christopher Clavius* hath, pag. 208 of his *Geometria practica*. I shall give a third in conformity to what follows.

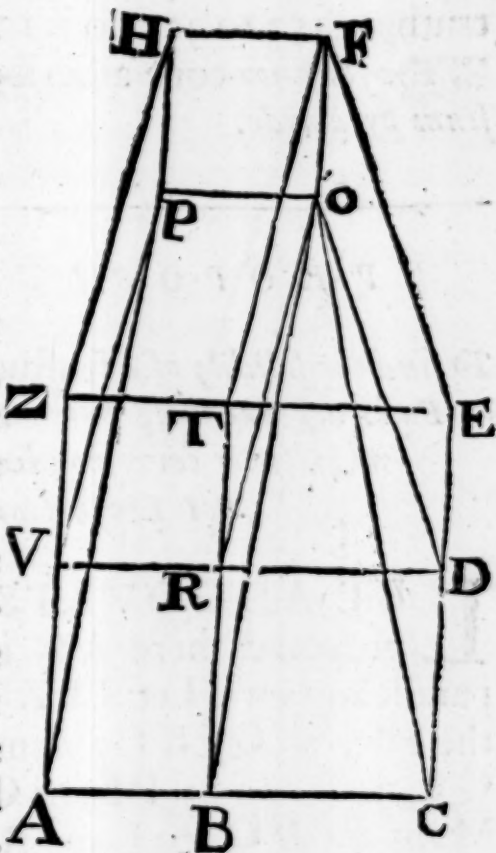
*A Third way.*

Let ACEZHPOF be a *frustum pyramide* the Bases Squares and Parallel ; Let HPOF be projected in the Base ACFZ, as ZVRT ; VR and TR be continued to D and B ; then will TD be equal to RA, and RC a Square ; most manifest it is, that the parallelepipedon ZVRTPOFH, more the prisme VABRO P, more the *pyramide* RBCDO, more the prisme RDETOF is equal to the said *frustum pyramide*.

For

For all the parts of any magnitude being taken, together are equal to the whole. *Axiom* 19. I.

The prisme  $VABROP$  is equal to the prisme  $RDETFO$ , because of equal bases and altitude, but the prism  $VABROP$  is half the parallelepipedon, made of the base  $VABR$ , and the altitude  $RO$ , by 28. 11. therefore the parallelepipedon made of the base  $ZABT$ , that is the Rectangle made of  $HF$  and  $ZA$ , and the altitude  $RO$ , more



the *pyramide*  $RBDRO$  that is  $\frac{1}{3}$  of the square of the difference of the sides of the upper and lower Bases and the altitude  $RO$ , shall be equal to the *frustum* proposed.

*Example in Numbers.*

$ZE$ , 60;  $ZT$ , 26; the product 1560 equal to  $BZ$ , a third part of  $RC$  is  $385\frac{1}{3}$ , which added

( 6 )

added to 1560, is equal to  $1945\frac{1}{3}$ , this last composed number multiplyed by 68 equal to R O makes  $132282\frac{2}{3}$ ; the solidity of the said *frustum*; then as 14 to 11, or more near the truth, as 452 to 355, so is  $132282\frac{2}{3}$  to 103894  $\frac{194}{339}$  the *frustum* cone adscribed within the *frustum* pyramide.

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P R O P O S I T I O N II.

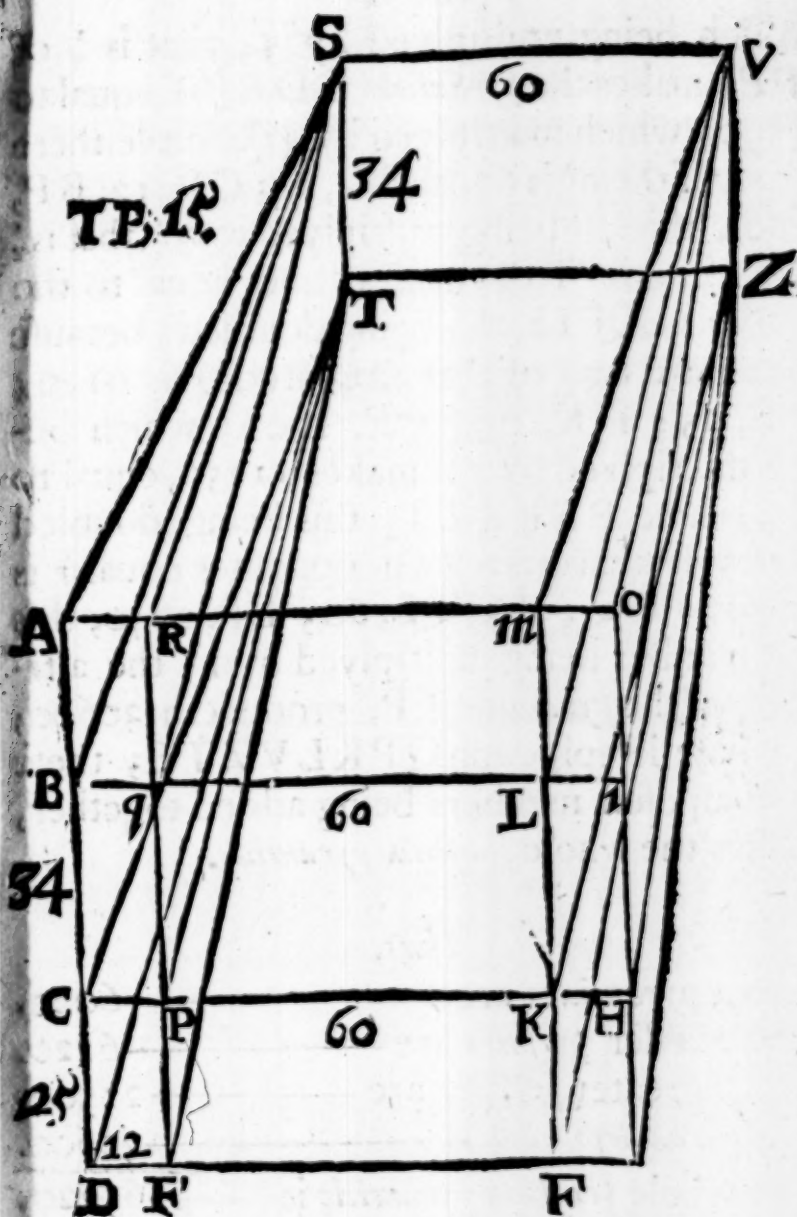
To find the solidity of a *frustum* pyramide, whose Bases are parallel, but not alike; that is, when their corresponding sides are not proportional.

LET ADGOVZTS be the *pyramide* proposed, here A G is a square, S Z a parallelogram; Let S T Z V be projected in the base, as, Q P K L; continue the sides, as Q L, to B and I; P K, to C and H; L K, to M and F; P Q, to E and R; the Figure being completed, the whole *frustum* will be composed of these parts, namely 4 *pyramides*, as C D E P T, K F G H Z, L I O M V, R Q B A S, more 4 *prismes*, as B C P Q T S, Q R M L V S, L K H I V Z, K F E P T Z, more the parallelepipedon P Q L K Z V S T; a further demonstration is needless.

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### An Example.

C D equal to 25, D E equal to 12, the  
like for the other Angles, C E equal to 300,  
B 4 which

which being multiplied by 5, that is  $\frac{1}{3}$  of TP, makes the *pyramide* CDEPT equal to 1500, which multiplied by 4 (because there are 4 of them) is 6000; BC, 34; CP, 12; BP, 108; this last being multiplied by  $7\frac{1}{2}$  that is the altitude TP, makes 3060 equal to the *prisme* BCQPTS, this being doubled (because there are two of that magnitude) is 6120; PE, 25; PK, 60; PF, 1500; which being multiplied by  $7\frac{1}{2}$  makes 11250, equal to the *prisme* PEFKZT, this being doubled (because there are two opposites equal) is 22500; PQ, 34; QL, 60; LP, 2040; this last number being multiplied by 15 the altitude of the *pyramide* TP, produceth 30600; the parallelepipedon QPKLVZTS; these composed numbers being added together, makes the whole *frustum pyramide*.

Thus.

|                                      |        |
|--------------------------------------|--------|
| The 4 <i>pyramides</i> are           | 6000.  |
| The 2 lesser <i>prismes</i> are      | 6120.  |
| The 2 greater <i>prismes</i> are     | 22500. |
| The <i>parallelepipedon</i> is       | 30600. |
| The whole <i>frustum pyramide</i> is | 65220. |

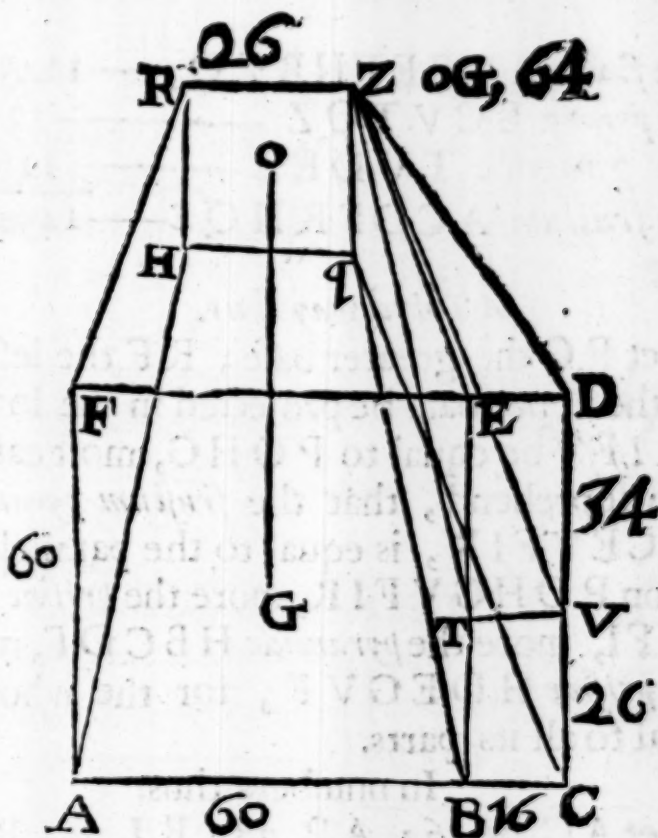
A Second way thus.

Let FC be the greater base, RQ the lesser, OO the height; make it as RH : HQ :: FA : AB; draw BE parallel to FA, then will the



(9)

the plane  $F B$  be like the plane  $R Q$ ; Let the Figure be completed, then will the *frustum*  $A C D F R H Q Z$  be equal to the *frustum*  $A B E F R H Q Z$  more the *prisme*,  $B C V T Q Z$ , more the *pyramide*  $T V D E Z$ : here note that  $T B$  is made equal to  $Q Z$ .



*The Example in Numbers.*

The *frustum*  $A B E F R H Q Z$ , is equal to  $12450 \frac{1}{3}$ , found by the first *Proposition*,  $B C$ ,  $16$ ;  $C V$ ,  $26$ ;  $B V$ ,  $416$ ; being multiplied by  $\frac{1}{3}$  the altitude, that is  $32$ , makes  $13312$ , equal

equal to the *prisme*  $BCVTQZ$ ;  $TV$ , 26;  
 $VD$ , 34;  $TD$ , 544; being multiplied by  
 $\frac{1}{3}$  of  $GO$  that is  $21\frac{1}{3}$ , makes  $11605\frac{1}{3}$  equal to  
the *pyramide*  $TVDEZ$ ; these three com-  
posed numbers being added together, makes  
 $149418\frac{2}{3}$  equal to the *frustum pyramide*  $ACD$   
 $FRHQZ$ .

|                               |                          |
|-------------------------------|--------------------------|
| The <i>frustum</i> $ABEFHRZQ$ | — 124501 $\frac{1}{3}$ . |
| The <i>prisme</i> $BCVTQZ$    | — 13312.                 |
| The <i>pyramide</i> $TVDEZ$   | — 11605 $\frac{1}{3}$ .  |
| The <i>frustum</i> $ACDFRHZQ$ | — 149418 $\frac{2}{3}$ . |

*A Third way thus.*

Let  $PC$  the greater base;  $RF$  the lesser;  
Let the upper base be projected in the lower,  
as  $RI FV$  be equal to  $POHG$ , most easie it  
is to apprehend, that the *frustum pyramide*  
 $PACEVFIR$ , is equal to the parallelepi-  
pedon  $POHGVFIR$ , more the *prisme*  $OA$   
 $BHFI$ , more the *pyramide*  $HBCDF$ , more  
the *prisme*  $HDEGVF$ , for the whole is  
equal to all its parts.

*In numbers thus.*

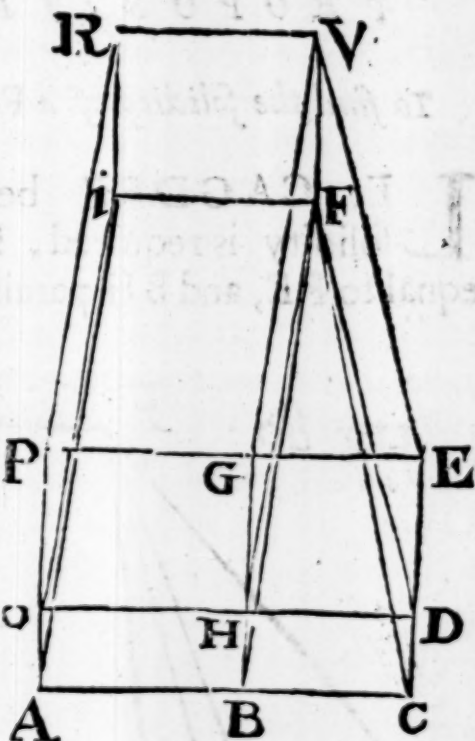
Let  $AC$ , be 56;  $AP$ , 38;  $RI$ , 26;  $RV$ ,  
30;  $RP$ , 40;  $RI$ , 26;  $VR$ , 30;  $IV$ , 780  
equal to  $PH$ ; which being drawn into  $RP$ ,  
40; makes  $POHGVFIR$ , 31200:  $OA$ ,  
12;  $AB$ , 30;  $AH$ , 360; being multiplied  
by 20, that is half the height  $PR$ , it makes

7200

(11)

7200 equal to OABHFI. BC, 28;  
HB, 12; BD, 336; being multiplied by  $\frac{1}{3}$

of the altitude  
that is  $13\frac{1}{3}$ , it  
produceth 4480,  
equal to the pyra-  
mide HBCDF;  
GH, 26; HD,  
28; HE, 728;  
being multiplied  
by 20, the product  
is 14560 equal to  
the prisme HDE  
GVF; these 4  
composed num-  
bers being added  
together, makes  
the solidity of the  
said frustum.



The parall. lepipeton-POHGFVRI=31200  
The prisme————OABHFI=07200  
The pyramide————HBCDF=04480  
The prisme————HDEGVF=14560

The frustum pyramide--PACEVFIR=57440

Then,

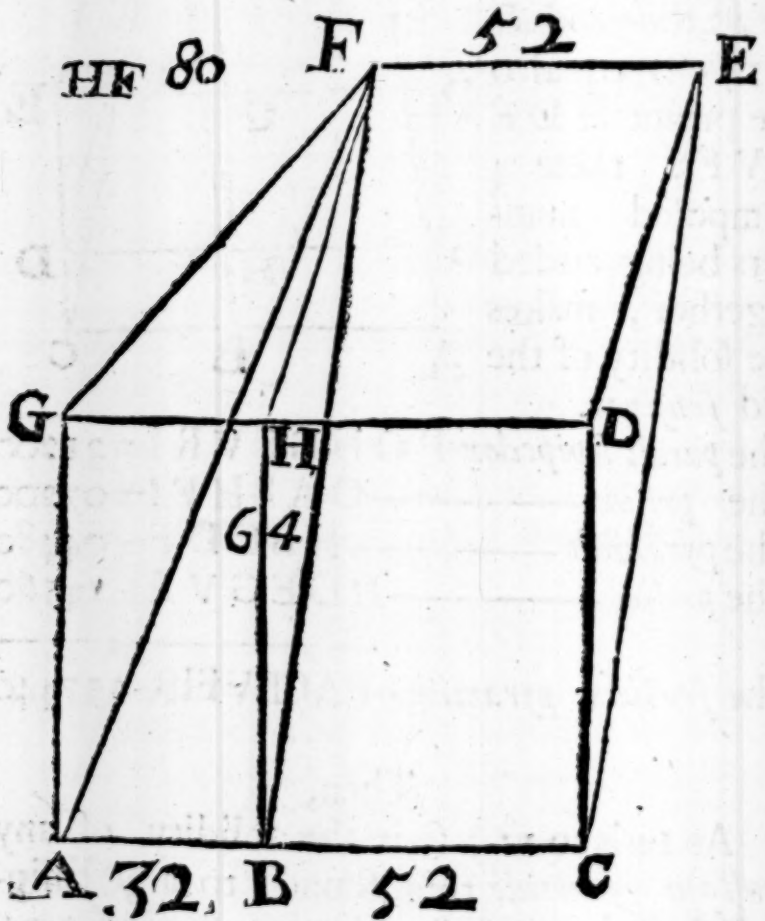
As 14 is to 11, so is the solidity of any  
frustum pyramide thus found, to any Ellip-  
tick frustum, adscribed in that frustum pyra-  
mide.

Proposition.

## PROPOSITION III.

To find the solidity of a Frustum Prisme.

**L** Et CAGDEF be a *prisme*, whose solidity is required, let BC be made equal to FE, and BH parallel to CD, and the



Figure

Figure being completed, the *frustum prisme* ACDGFE may be composed of the *prisme* BCDHFE, more the *pyramide* BAGHF: a further demonstration is needless. 

*An Example in Numbers.*

BC, 52; BH, 64; BD, 3328; this last number being multiplied by 40, that is half the height, makes 133120 equal to the *prisme* HBCDEF; AB, 32; BH, 64; BG, 2048: this last being drawn into  $26\frac{2}{3}$ , that is, a third part of the altitude makes 54613 $\frac{1}{3}$  equal to the *pyramide* GABHF: this *prisme* and *pyramide* being added together, makes 187733 $\frac{1}{3}$ , equal to the *frustum prisme* AGDCEF.

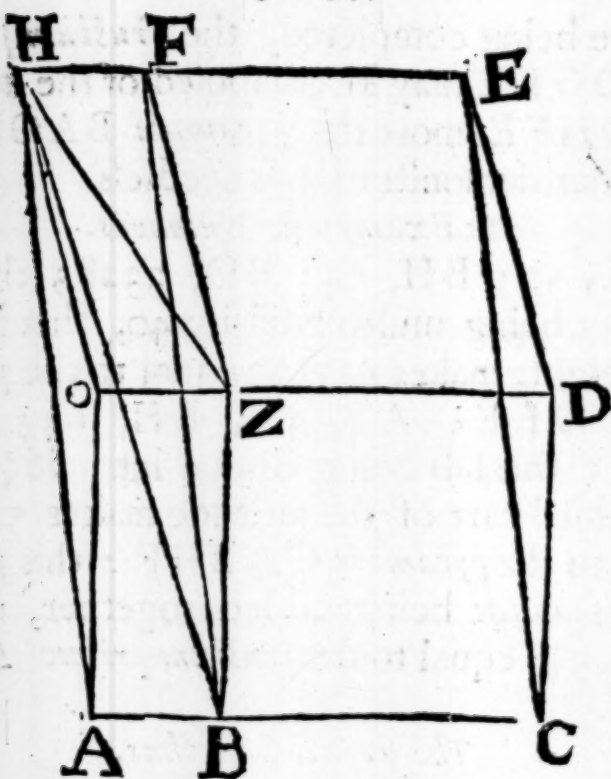
*The second Case thus.*

Let ZBCDEH be a *frustum prisme*; Let AC be equal to HE and AO parallel to ZB, the Figure being completed, the *prisme* ACDOHE less the *pyramide* ABZOH is equal to the *prisme* BCDZHE; or thus, the *prisme* ZBCDFE more the solidity ZBHF, that is, half the *pyramide* ZBAOH, is equal to the *frustum* proposed.

*Example in Numbers.*

AC, 84; AO, 64; AD, 5376; this last drawn into 40, half the height makes 215040; equal to the *prisme* OACDHE; AC, 32; BZ, 64; AZ, 2048 being multiplied by  $26\frac{2}{3}$ , that is, one third of the height, it makes





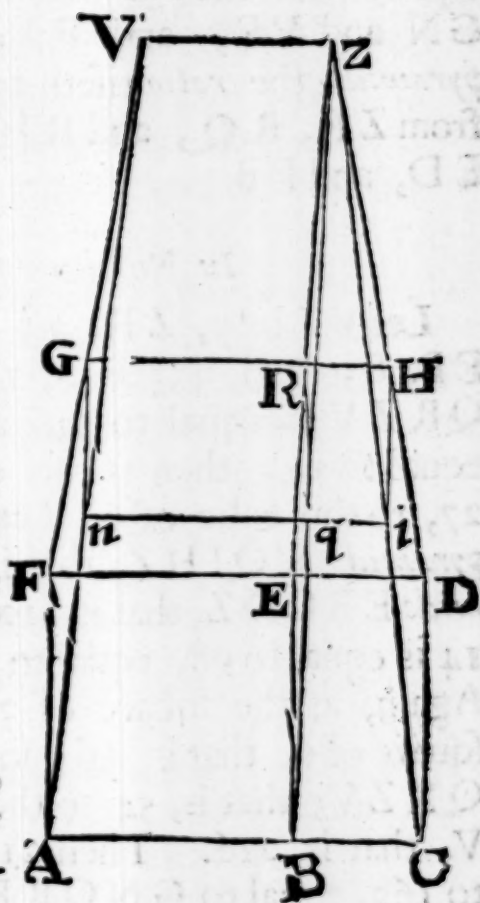
54613  $\frac{1}{3}$  equal to the *pyramide* A B Z O H, O H, this *pyramide* taken from the *prisme*, leaves 160426  $\frac{2}{3}$ , equal to the *frustum prisme* proposed.

Then as 14 is to 11, so are such *prismes* to *Elliptick solids* adscribed in those *prismes*.

Hence follows a *Fourth way* for finding the *Solidity* of these *irregular frustums*.

Suppose A C D F G N I H be the *frustum* proposed, because F A is equal to D C, and A C equal to D F, and G N equal to I H, and N I

NI to GH; and because GN is less than FA, and HI than DC; therefore if AN, CI, DH, FG, be continued towards Z and V, they will meet, suppose at Z and V, then ACDFVZ, less NIHGVZ, there remains ACDFGNIH the *frustum* proposed.



It also follows, That such *prismes* have such proportions one to another, as Squares and Cubes of their corresponding sides, disjoyned, thus.

The *pyramide* BCDEZ, is to the *pyramide* RQIHZ, as the cube of BE to the cube of RQ; 8, 12. The *prisme* FAEVZ, is to the *prisme* GNQRVZ, as the square of FA, is to the square of GN; the reason why these *prismes* are not in a triplete *ratio* of their homologal sides, as well as the *pyramides*; is because FE, GR, and VZ are equal,

equal, and the *ratio* riseth but from V G, G N and V F, and F A; whereas in the *pyramide* the *ratio* riseth from three, that is, from Z R, R Q, and R H; and from Z E, E D, and E B.

*In Numbers thus.*

Let V Z be 9, Z R, 3; Z E, 6; R Q, 4; E B, 8; R H, 3; E D, 7; Therefore G N Q R Z V is equal to 54, and R Q I H Z is equal to 14; then as the cube of 3, that is 27, to the cube of 6, that is 216; so is the *pyramide* R Q I H Z, that is 14, to the *pyramide* E B C D Z, that is 112. Then 112 less 14 is equal to 98, equal to R Q I H D C B E. Again, as the square of 3, that is 9, to the square of 6, that is, 36; so is the *prisme* G N Q R Z V; that is, 54 to the *prisme* F A B E Z V, that is, 216. Then 216 less, 54 is equal to 162, equal to G N Q R E B A F. This last *frustum*, more the *frustum* R Q I H D B C E is equal to the *frustum* G N I H D C A F equal to 260.

*Or thus.*

As 27 : 189 :: (that is the cube of Z E less the cube of Z R) 14 : 98, equal to the *frustum* R Q I H D C B E. Again, as 9 : 27 :: (that is the square of Z E less the square of Z R) 54 : 162, equal to the *frustum* G N Q R E B A F.

What

What ever is said of these *prismes* and *pyramides*, the same is to be understood in Cones and Ecliptick *prismes* ;

For they are in duplicate and triplicate *ratio* of their homologal sides, further for as much as that, the two first terms in each proportion may be fixt; there may by the help of a Table of Squares and Cubes, the solidity of such solids easily be calculated gradually, that is, inch by inch, or foot by foot, the two fixed numbers in the *pyramide* are the Cube of the continuation, or the side R Z; and the *pyramide* R Q I H, the like in the *prisme*, and with the square of the continuation R Z.

Thus.

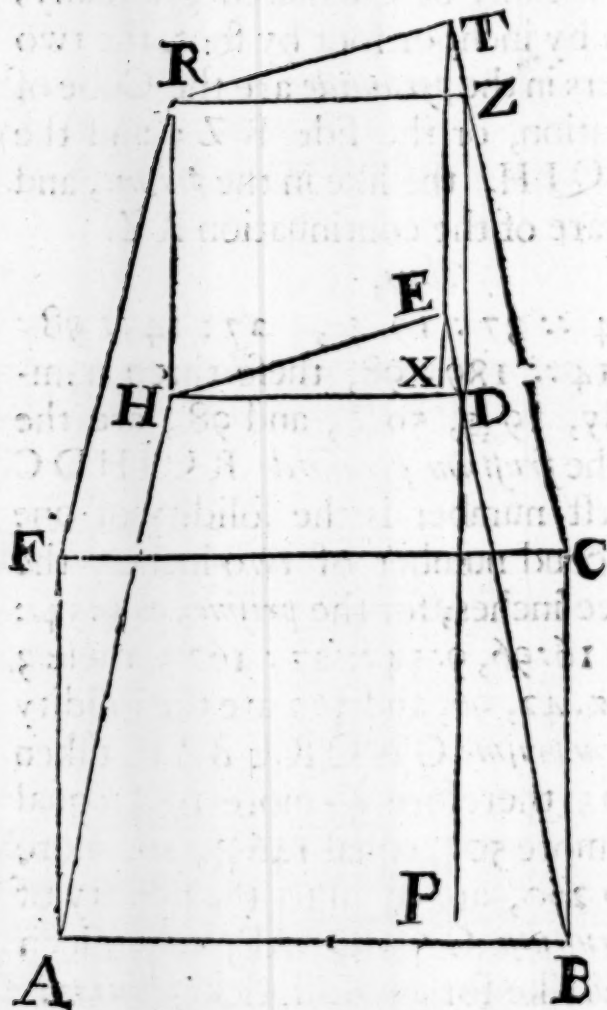
27 : 14 :: 37 : 19  $\frac{5}{17}$ , 27 : 14 :: 98 : 50  $\frac{22}{17}$ , 27 : 14 :: 189 : 98, these three numbers, namely, 19  $\frac{5}{17}$ , 50  $\frac{22}{17}$ , and 98, are the solidity of the *frustum pyramide* R Q I H D C B E, the first number is the solidity of one inch, the second number of two inches, the third of three inches, for the *prisme*; as 9:54:: 7:42, 9:54:: 16:96, 9:54:: 27:162; these 3 numbers, viz. 42, 96, and 162 are the solidity of the *frustum prisme* G N Q R E B A F, taken inch by inch; therefore 42 more 19  $\frac{5}{17}$  equal to 61  $\frac{5}{17}$ , 96 more 50  $\frac{22}{17}$  equal 146  $\frac{22}{17}$ , 162 more 96 equal to 258, are equal to the solidity of the whole *frustum* G N I H D C A F, taken inch by inch; the like for any Elliptick *frustum*.

C

Proposition.

Proposition. IV.

To find the solidity of a frustum pyramide,  
whose Bases are not parallel; and the incli-  
nation is from side to side.



**L** Et R  
H E  
TCBAF  
be a fru-  
stum pyra-  
mide, the  
base RH  
ET, not  
parallel to  
the base F  
ABC; let  
RH DZ  
be parallel  
to ABC  
F, then  
may the  
whole fru-  
stum pyra-  
mide RH  
ETABC  
F be com-  
posed of  
the fru-  
stum

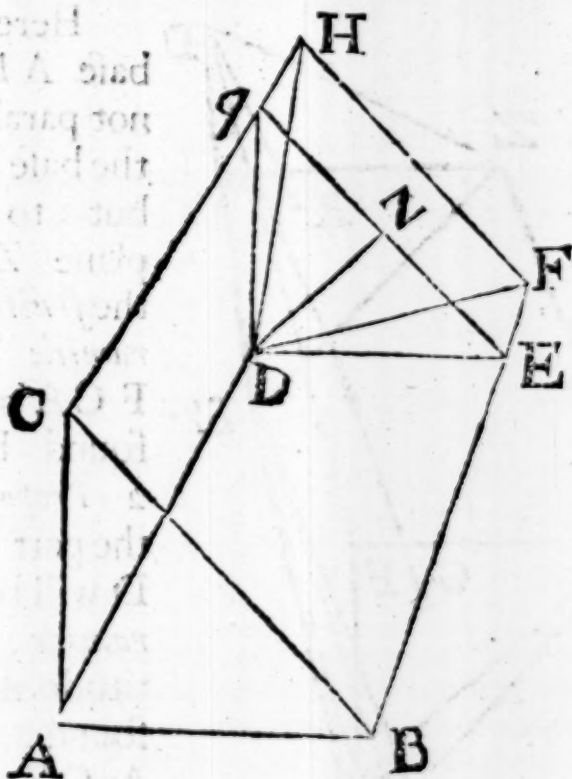


( 19 )

*stump pyramide*  $RHDZABCF$  more, the  
*prisme*  $RHDZTE$ , by a composition of  
 the last *Propos.*  $PD$  being the altitude of the  
*frustum pyramide*  $RHDZABCF$ :  $EX$  the  
 height of the *prisme*  $RHDZTE$  may be  
 found, thus.  $BD:DP::DE:EX$ ; a fur-  
 ther demonstration is needless.

II.

To find the solidity of a triangular frustum  
 pyramide, whose bases are not parallel.



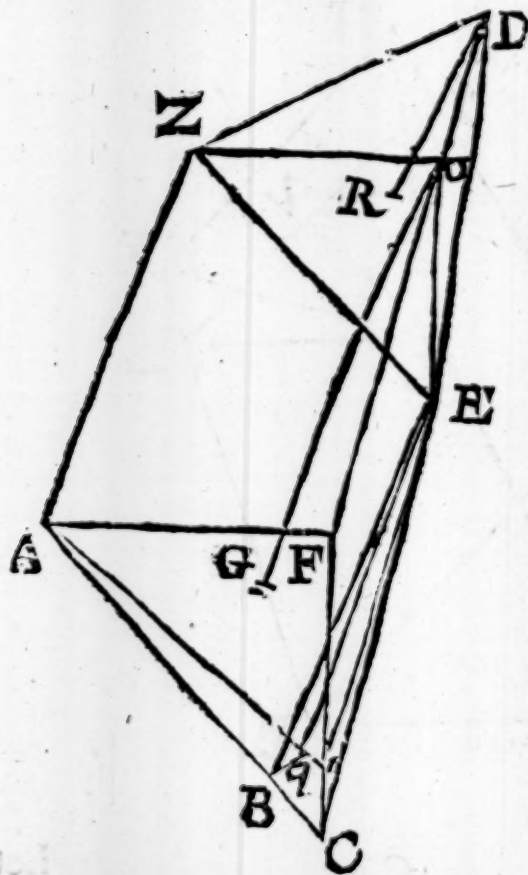
C 2

Let

Let  $CABDHF$  be a *frustum pyramide*,  $CAB$  not parallel to  $HDF$ , let  $QDE$  be parallel to  $CAB$ ; the triangular *frustum*  $QDEBAC$  is half a quadrangular *frustum pyramide*, found by the 2 *Proposition*,  $DZ$  the height of the *pyramide*  $QHFED$ ; Therefore the *frustum pyramide*  $QDEBAC$  more the *pyramide*  $QHFED$  will be equal to the *frustum pyramide*  $FHDABC$ .

## III.

To find the solidity of the *frustum pyramide*,  $ACFEDZ$ .



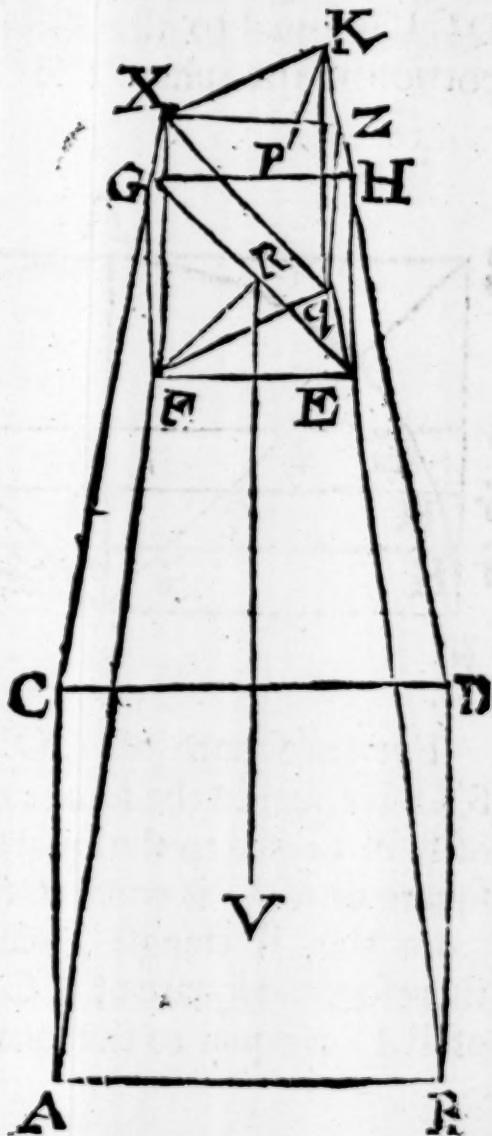
Here the base  $AFC$  is not parallel to the base  $ZDE$  but to the plane  $ZOE$ ; the *frustum pyramide*  $ZOE FCA$  may be found by the 2 *Proposition*, the part  $ZOE D$  will be a *pyramide*, its altitude may be found, thus, As  $OF : OG :: OD : DR$ , then

then the *frustum pyramide* ZOEFCA more  
the *pyramide* ZOED will be equal to the  
solid ACFE OZD. IV.

#### IV.

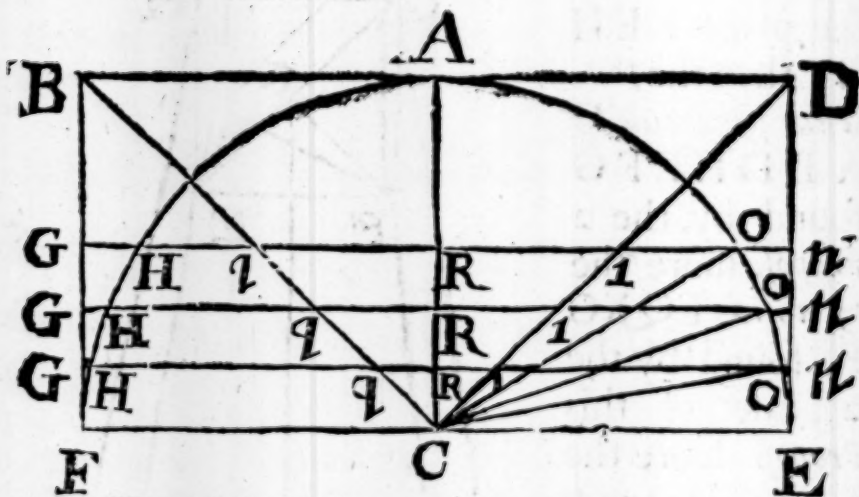
*To find the solidity of a frustum pyr2.mide, whose bases are not parallel, and whose inclination is from Angle to Angle.*

Let  $CABDFXKQ$  be a *frustum pyramide*, the base  $CABD$  not parallel to the base  $FXKQ$ , but parallel to the plane  $FEHG$ ; then the *frustum pyramide*  $CABDHEFG$  found by the 2 *Propos.* more the *pyramide*  $EQXG$   $F$ , found by the 2 case of this *Propos.* more the *frustum pyramide*  $GEHZXQ$ , found by the 2 *Propos.* more the *pyramide*  $XKZQ$ , found by the 3 case of this *Propos.* will be equal to the whole *frustum pyramide*  $CABDFXKQ$ .



## P R O P O S I T I O N V.

**L** Et F H A O E be a Semicircle, H O parallel to F E ; F B and E D parallel to C A, the Figure being completed, the *Cone* Q C I is equal to the *Cylinder* G E less the portion of the Sphere F H O E.

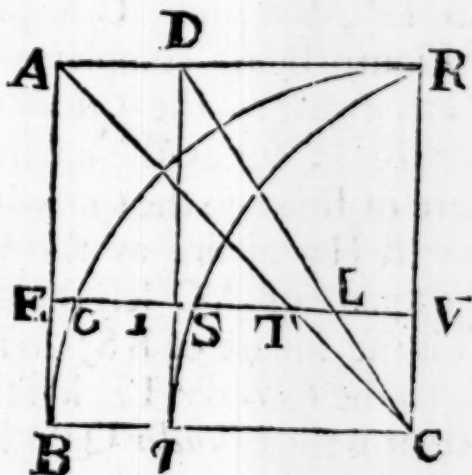


For the square of C O less the square of R O is equal to the square of R C, the square of R N (equal to the square of C O) less the square of R O is equal to the square of O N more the Rectangle twice under R O N; therefore the square of R C (that is the square of R I) is equal to the square of O N more, the

the double Rectangle  $RON$ , by  $4:2$ . every one of these being considered one by one; but being collected (*viz.*) all the squares of  $RI$  is equal to all the squares of  $RN$ , less all the squares of  $RO$ ; or if there be taken the quadruple of them, all the squares of  $QI$  is equal to all the squares of  $GN$  less all the squares of  $HO$ : by  $2:12$ , all the circles; therefore the *Cone*  $QCI$  is equal to the *Cylinder*  $GE$  less the portion of the Sphere  $HFE O$  which was proposed.

*The second Case.*

Let  $ROBC$  be  $\frac{1}{4}$  of a Circle, and  $RSQC$   $\frac{1}{4}$  of an Ellipsis complete the Diagram, then will the square of  $QC$ , that is  $IV$  less the



$C 4$

squares



square of  $LV$  be equal to the square of  $SV$ . For  $AR:RD::TV:VL$ , therefore the squares of them, and  $AR:RD::OV:VS$ , therefore the squares of them; therefore, as the square of  $TV$ , to the square of  $VL$ , so the square of  $OV$ , to the square of  $VS$ ; Therefore, as the square of  $EV$ , to the square of  $EV$  less the square of  $TV$ . So the square of  $IV$ , to the square of  $IV$  less the square of  $LV$ , but the square of  $EV$  less the square of  $TV$  is equal to the square of  $OV$ ; therefore the square of  $IV$  less the square of  $LV$  equal to the square of  $SV$ .

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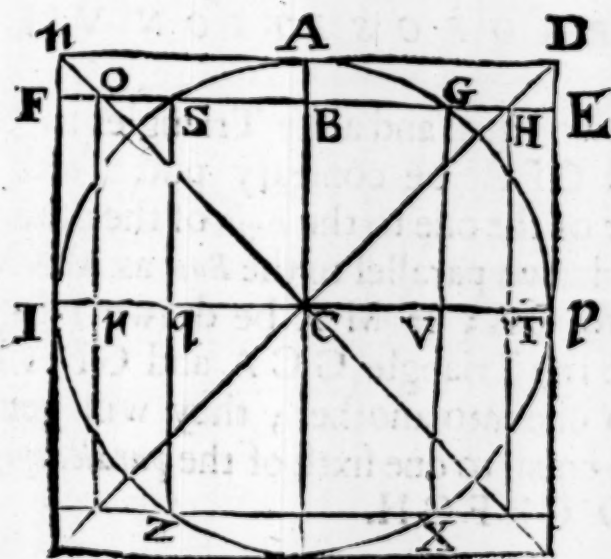
P R O P O S I T I O N VI.

**I**N the Hemisphere  $IAP$ , Let  $FE$  be parallel to  $IP$ ,  $SZ$  and  $GX$  parallel to  $AC$ ; the Figure being completed; the *Cylinder*  $QG$  is equal to the *Cylinder*  $IE$  less the *Cylinder*  $RH$ ; for as the square of  $AC$ , to the square of  $BC$ ; so the *Cylinder*  $IE$ , to the *Cylinder*  $RH$ ; again, as the square of  $AC$ , to the square of  $AC$  less the square of  $BC$ , (that is the square of  $BS$ ) so is the *Cylinder*  $IE$ , to the *Cylinder*  $IE$  less the *Cylinder*  $RH$ , (that is the *Cylinder*  $QG$ ) it followeth that the *Cylinder*  $RH$  is equal to the excavatus *Cylinder*  $IQSFGE$   $VPV$ , for the  
cylind-

*Cylinder QG* is equal to the excavetus *Cylinder* *IROFHTPE*.

But if the excavetus *Cylinder* *ORQSGVTH*, be added to both; the *Cylinder* *RH* will be equal to the excavetus or hollow *cylinder* *QIFSGVPE*; but the cone *OCH* is equal to *FISGPE* by the first part of the Fifth Proposition; Therefore *IQSGVP* is equal to *ORCTH*, that is,  $\frac{1}{3}$  of the difference betwixt the circumscribed *IE* and inscribed *QG* cylinders; subtracted from the circumscribed *cylinder* *IE* is equal to the portion of the Sphere *ISGP*; or if  $\frac{1}{3}$  of the difference be added to the inscribed *cylinder* *QG* it will be equal to the portion of the Sphere *ISGP*.

The like in a Spheroid by the second part of the last Proposition.



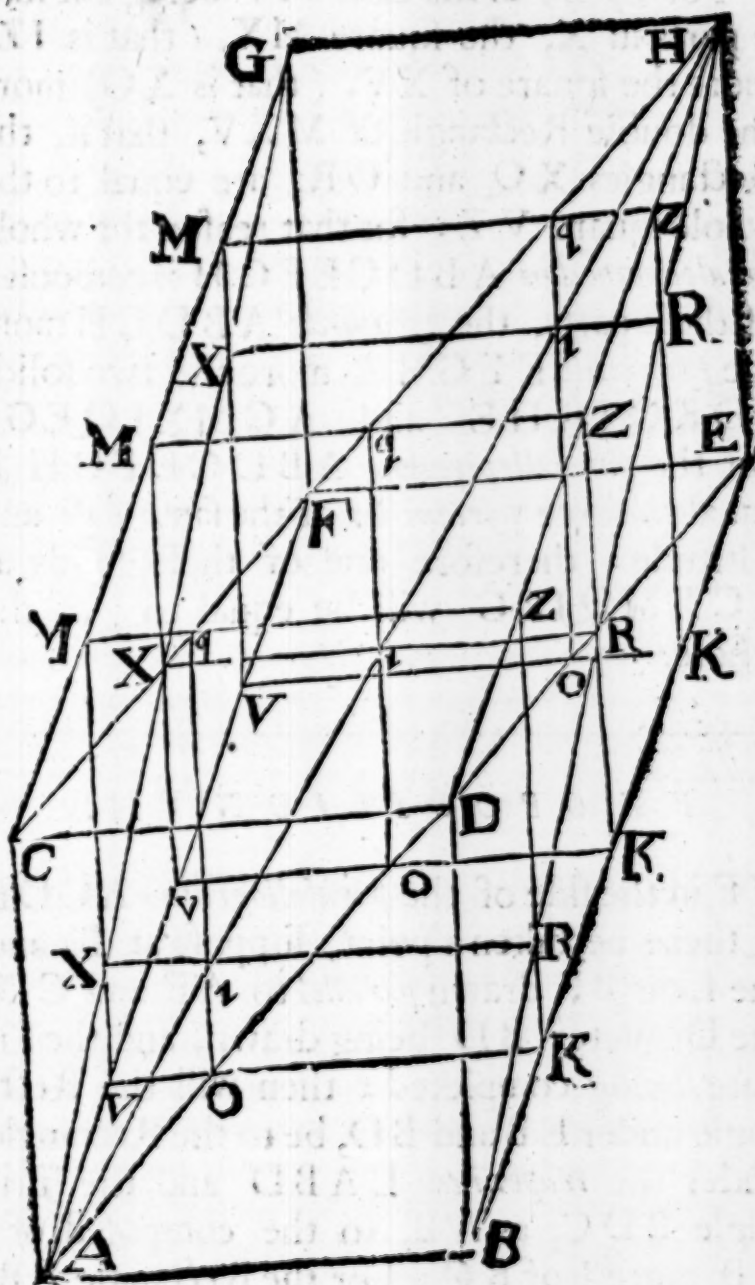
*In Numbers thus.*

Let  $IP$  be 20;  $QV$ , 16;  $CB$ , 10; the square of  $IP$  is 400, the square of  $QV$  256; the difference of the squares is 144, a third part of that number is 48, which subtracted from the greater square 400, leaves 352: or if two thirds of the difference, that is, 96 be added to the lesser square, that is, to 256, the sum will be 352 the true mean square; but if it be as  $14 : 11 :: 352 : 276$   $\frac{1}{2}$  the mean circle, which being multiplied by the length  $ZS$ , 20; the Product is  $5531\frac{1}{2}$  the solidity of the portion  $ZISG$   $PX$ .

## P R O P O S I T I O N VII.

**I**F two equal and alike Triangles as  $GCA$  and  $GFA$  be contrary put, (*viz.*) the *Vertex* of the one to the *Base* of the other, and Lines drawn parallel to the *Base* as  $MXV$  parallel to  $CA$ ; If  $MX$  be drawn into  $XV$ , that so the Triangle  $GCA$  and  $GFA$  being drawn one into another; they will generate a solid equal to one sixth of the *parallelepipedon*  $ABDCEFGH$ .

For



For

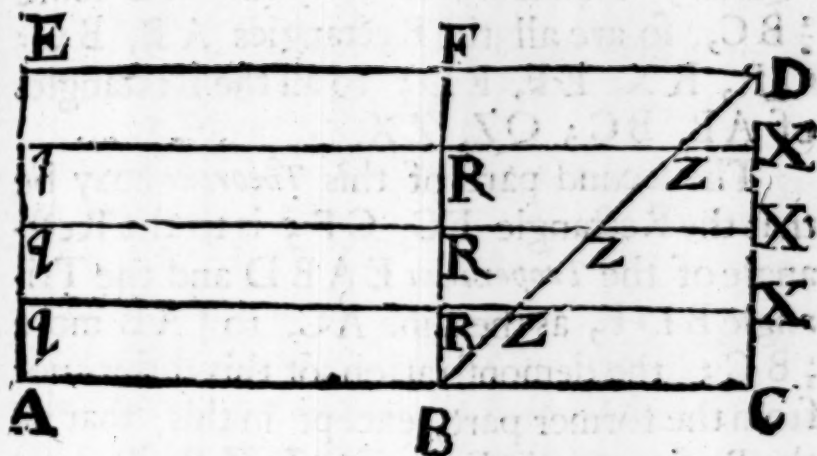
For by 4; 2. the Line  $MV$  being cut into 2 parts in  $X$ , the square  $MX$ , (that is  $IZ$ ) more the square of  $XV$ , (that is  $XO$ ) more the double Rectangle of  $MXV$ , that is, the Rectangles  $XQ$  and  $OR$ , are equal to the whole square  $VZ$ ; for that reason the whole *parallelepipedon*  $ABDCEFGH$  is composed of these parts, the *pyramide*  $ABDCH$  more the *pyramide*  $EFGH A$  more the two solids  $BDKOIRHE$  and  $ACMXIQFG$ , but the *parallelepipedon*  $ABDCEFGH$  is equal to three *pyramides* of the same *Base* and *Altitude*; therefore one of these solids as  $ACXMQIFG$  will be equal to  $\frac{1}{3}$  of the whole.

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P R O P O S I T I O N V I I I .

**I**F in the side of the *parallelogram*  $ACDE$  there be taken a point, suppose at  $B$ , and the Line  $BF$  drawn *parallel* to  $AE$  and  $CD$ , the Diameter  $BD$  being drawn, and the Figure being completed; then will the Rectangle under  $EB$  and  $BD$ , be to the Rectangle under the *trapezium*  $EABD$  and the Triangle  $BDC$ , as  $AB$ , to the composed of  $\frac{1}{2}$   $AB$  more  $\frac{1}{2}$  of  $BC$ ; For the Rectangle  $EB, BD$ ; that is, all the Rectangles of  $AB, BC; QR, RX; EF, FD$ ; that is the *parallelepipedon*





pipedon made of  $AB$ ,  $BC$ ; and the Altitude  $BF$ : the Rectangle  $EABD$ ,  $BDC$ ; that is, all the Rectangles  $EF$ ,  $FD$ ;  $QR$ ,  $RZ$ ; that is a *prisme* of  $EFD$  and the Altitude  $FB$ ; more all the Rectangles of  $RZ$ ,  $ZX$ ; but a *parallelepipedon* is to a *prisme* of the same Base and Altitude as  $1$  to  $\frac{1}{2}$ , that may be as  $AB$ , to  $\frac{1}{2} AB$ , but all the Rectangles of  $RZ$ ,  $ZX$ , are  $\frac{1}{2}$  of the *parallelepipedon*  $BCB$  and the Altitude  $CD$  by the last Proposition; but the *parallelepipedon*  $BCBFD$ , is to  $BC$ , as  $\frac{1}{2}$  of the *parallelepipedon*  $BCBFE$ , to  $\frac{1}{2}$  of  $BC$ , the *prisme* made of  $DFE$  and the Altitude  $EA$ , more all the Rectangles of  $RZ$ ,  $ZX$ ; that is  $\frac{1}{2}$  of the *parallelepipedon*  $BCB$  and the Altitude  $BF$ , is equal to the Rectangle  $EABD$ ,  $BDC$ ; that is all the Rectangles of  $QZX$ ;

$QZX$ ; Therefore as  $AB$ , to  $\frac{1}{2} AB$  more  $\frac{1}{2} BC$ , so are all the Rectangles  $AB$ ,  $BC$ ;  $QR$ ,  $RX$ ;  $EF$ ,  $FD$ ; to all the Rectangles of  $AB$ ,  $BC$ ;  $QZ$ ,  $ZX$ .

The second part of this *Theorem* may be that the Rectangle  $EC$ ,  $CF$ ; is to the Rectangle of the *Trapezium*  $EABD$  and the Triangle  $BD F$ , as the Line  $AC$ , to  $\frac{1}{2} AB$  more  $\frac{1}{2} BC$ ; the demonstration of this differs not from the former part, except in this, that *all* the Rectangles  $DFD$ ,  $ZRZ$ ,  $ZRZ$ , are  $\frac{1}{2}$  of the *parallelepipedon*, as appears by the last Proposition.

This Proposition is the 30 of the second of *Caval. Geomet. Indivis.* and is of good use in *Solid Geometry*.

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### PROPOSITION. IX.

**L**ET  $ADBEK$  be a *cone* and be cut through the *axis*, whose Section will be the Triangle  $AKB$ , Let it be cut by another Plane as  $EZD$ , the diameter of the Section  $ZB$  parallel to  $BK$ , the *Base* of the Section  $ED$  at Right Angle at  $C$  with  $AB$ ; Let it be as  $ABA : AKB :: PZ : ZK$ , then will  $PZC$  be equal to the square of  $CE$ ;

For,



if the *semiparabola* EIZRC be moved about the *axis* ZC, until it return to that point from whence it began its motion, it will *Generate* a *solide*, *Archimedes* names such a solid, a Rect-angled *conoide*; but generally it is called by the name of a *parabolical conoide*.

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### P R O P O S I T I O N X.

**L**et AVBXK be a *cone*, and be cut through the *axis*, whose Section shall be the Triangle AKB, Let it be cut by another Plane as NIEC whose diameter is NC, (being continued if need be) shall cut both sides of the *cone*, the Plane NIEC being at Right Angle with the Plane AKB, the Planes FED and TIG parallel to the Plane AVBX, Let it be made, as  $CRN:GRT::CN:NP$ , then will QRN be equal to the square of RI; for  $CN:NP::CR:RQ$ , taking the Altitude RN it may be  $CN:NR::CRN:QRN::CRN:GRT$ , in these last four terms CRN is twice, therefore QRN is equal to GRT, that is the square of RI.

By reason of the similitude of the Triangle CHD to CRG, and of the Triangle NRT to NHF, it may be, as

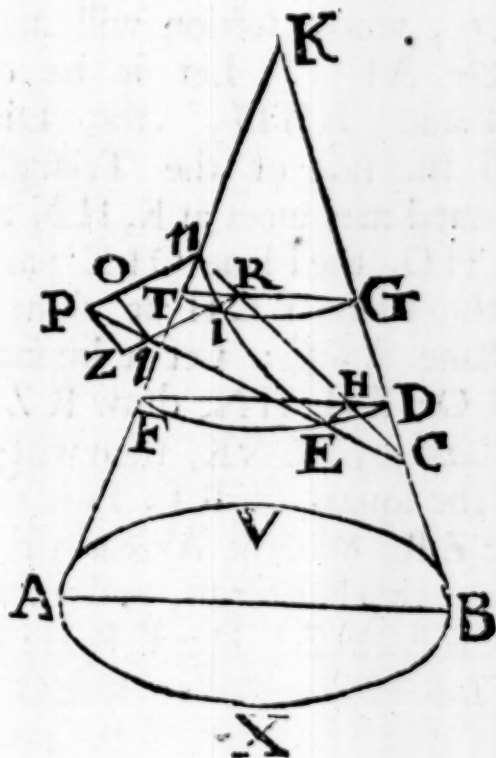
NR

(33)

$$\left. \begin{array}{l} \text{NR} : \text{TR} :: \text{NH} : \text{FH} \\ \text{RC} : \text{RG} :: \text{HC} : \text{HD} \end{array} \right\} \text{by 23. 6.}$$

$\text{NRC} : \text{TRG} :: \text{NHC} : \text{FHD}$ , but  $\text{TRG}$  is equal to the square of  $\text{RI}$ , and  $\text{FHD}$  is equal to the square of  $\text{HE}$ ; therefore as  $\text{NRC}$  to the square of  $\text{RI}$ , so is  $\text{NHC}$  to the square of  $\text{HE}$ .

This Section of a *cone* is named by *Apollo-*  
*nus* an *Ellipsis*.



D

But



But if the *Semi-Ellipsis* CEIN be moved about the *axis*, CN, until it return to that point from whence it began its motion, it will Generate a solid, *Archimides* calls such a solid a *Spheroides*; but if it be turned about the lesser Diameter, then he calls that solid a *Spheroides prolatus*.

P R O P O S I T I O N X I.

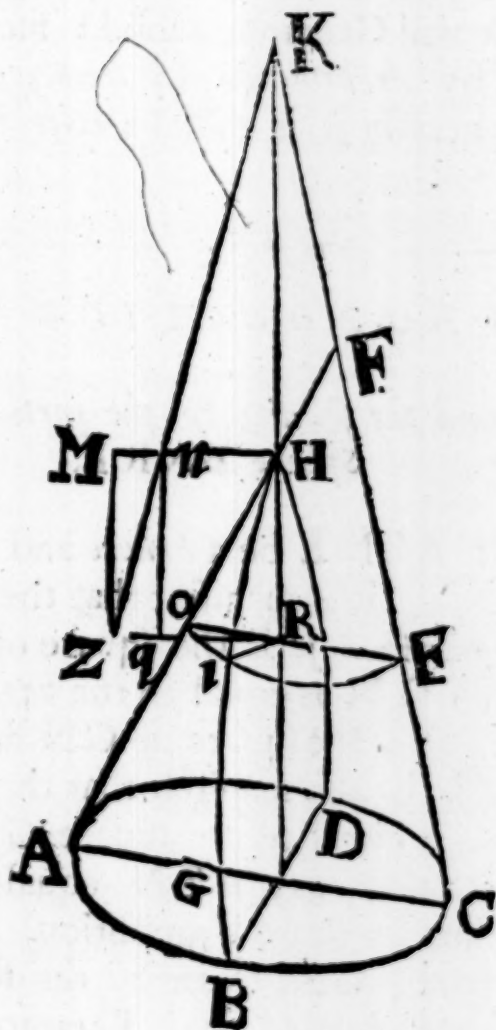
**L** Et ABCBF be a *cone*, and be cut through the *axis*, whose section will make the Triangle AFC, Let it be cut by another Plane BHD, the Diameter HG, and the side of the Triangle CF being continued may meet at K, HN at right Angle with HG, the Plane OIE parallel to ABCD, the Plane BDH at right Angle with the Plane AFC; Let it be made, as  $KRH:ERO::KH:HN$ , draw RZ parallel to HN, and joyn ZNK, then will ZRH be equal to the square of RI; For, as  $KH:NH::KR:ZR$ , take the Altitude RH and apply it to the two last terms, and it will be as  $KH:NH::KRH:ZRH::KRH:ERO$ , Therefore ZRH is equal to ERO that is the square of RI because of the similitude of the Triangle AGH to HRO, and of KGC to KRE, it may be as,

KG:

( 35 )

$KG:CG::KR:ER$  }  
 $GH:GA::RH:RO$  } by 23, 6.

$KGH:CGA::KRH:ERO$ ;  $CGA$  is equal  
to the square of  $GB$ , and  $ERO$  is equal to



the square of  $RI$ ; Therefore as  $KGH$ , to the square of  $GB$ , so is  $KRH$ , to the square of  $RI$ .

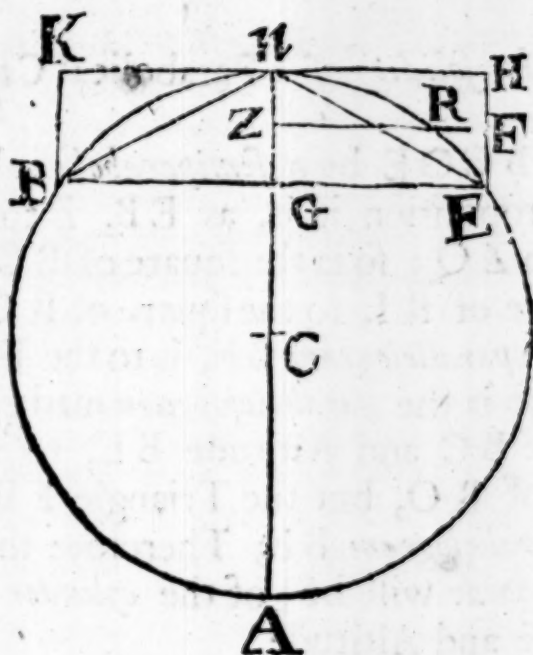
This manner of Section *Apollonius* calls a *Hyperbola*; but if the *semihyperbola*  $BIHRG$  be moved about the Axis  $HG$ , until it returns to that point from whence it began its motion, it will Generate a solid; such a solid is called by *Archimedes* an *Amblygon conoide*; but generally it is called a *Hyperbolical conoide*.

## P R O P O S I T I O N XII.

*To find the solidity of the portion of the  
Sphere  $BNRE$ .*

**L** Et  $ABNE$  be a *Sphere* and  $BH$  a *Cylinder*, it is manifest that the *parallelepipedon* whose *Base* is the square of  $GE$ , and *Altitude*  $GN$  is equal to the *parallelepipedon* whose *Base* is the square of  $GN$  and *Altitude*  $AG$ ; it is also manifest that the *Rectangle*  $AGN$  is equal to the square  $GE$ , the like for any other, as,  $AZN$  equal to  $ZRZ$ , Therefore by the 8 Proposition, as  $AG$  to  $AG$  more  $\frac{1}{2} GN$ , so are all the squares  $ZF$ , to all the squares of  $ZR$ ; Therefore, as  $AG$ , to  $\frac{1}{2} AG$  more  $\frac{1}{2} GN$ , so the *Cylinder*  $EK$ ,  
to

to the portion of the Sphere E R N B. The  
like in a *Spheroid*.



*In Numbers thus.*

Let AG be 150; GN, 34; GE 90; the *parallelepipedon* made of the square of BE and Altitude GN, is 1749600; then, as AG, 150; to  $\frac{1}{2}$  AG, more  $\frac{1}{6}$  GN; that is 75 more 9; that is 84; so is 1749600, to 979776, that is all the squares in the portion of the Sphere BNR E; then as 14 : 11 :: 979776 : 769824, the portion of the Sphere BNR E.

D 3

## Propoſ





*Thus in Numbers.*

Let AB, 10; BC, 10; Therefore AC, 20; Let BE, 30; the square of AC, 20, is 400, which being multiplyed by  $\frac{1}{2}$  EB, that is by 15; the Product is 6000, equal to all the squares in the *conoide* AEC; but if it be made as 14 : 11 :: 6000 : 4714  $\frac{1}{2}$  the solidity of the *conoide* AEC.

PROPOSITION. XIV.

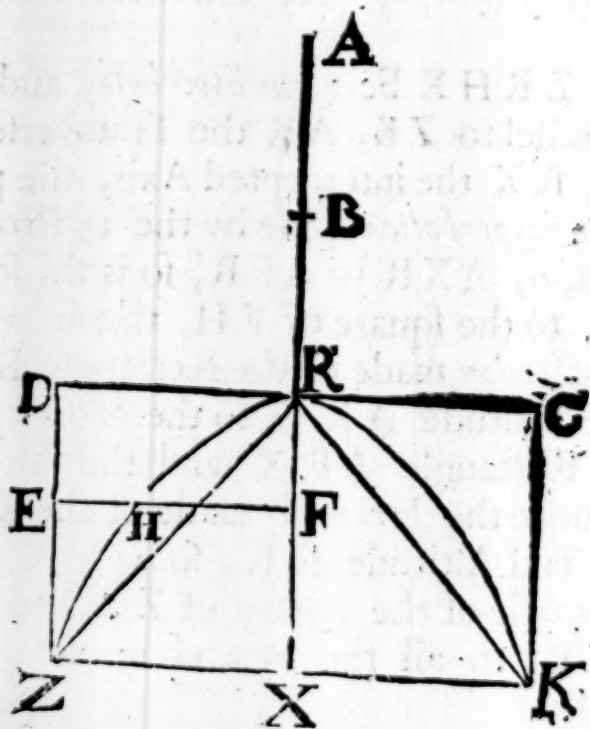
*To find the solidity of a Hyperbolical Conoide.*

Let ZRHX be a *semihyperbola*, and EF parallel to ZK, AR the Transverse Diameter, RX the intercepted Axis, the point H in the *Hyperbolical* Line by the 11 Proposition it is, as AXR to AFR, so is the square of XZ, to the square of FH, that is, as the *parallelepipedon* made of the Rectangle RXZ and the Altitude AX, is to the *prisme* made of the Rectangle ARX and the Altitude RD, more the *pyramide* made of the square of ZD and Altitude DR; so is the *parallelepipedon* made of the square of ZK and Altitude XR, to all the squares in the *conoide* ZHRK.

D 4

But

But as the *parallelepipedon* made of the Rectangle  $R X Z$  and Altitude  $A X$ , is to the *prisme* made of the Rectangle  $A R X$  and Altitude  $R D$ , more the *pyramide* made of the square  $D Z$  and Altitude  $D R$ ; so is the Line  $A X$  to the Line  $B R$  (that is  $\frac{2}{3} A R$ ) more  $\frac{1}{3}$  of the Line  $R X$ , by the 8 Proposition; Therefore by the 11. 5. it will be as the Line  $A X$ , is to the Line  $B R$ , more  $\frac{1}{3}$  of the Line  $R X$ , so is the *parallelepipedon* made of the square of  $Z K$  and Altitude  $X R$ , to all the squares in the *conoide*  $Z H R K$ ; then as 14:11, so are all the squares in the *conoide*  $Z H R K$ , to the *conoide*  $Z H R K$ .



Let

*In Numbers thus.*

Let  $AX$  be 150;  $RX$ , 54;  $ZX$ , 90;  $AR$ , 96;  $BR$ , 48; the *parallelepipedon* whose *Base* is the Rectangle of  $RXZ$  and the *Altitude*  $AX$  is 729000.

The *prisme* whose *Base* is the Rectangle  $ARX$  and *Altitude*  $RD$  is 233280.

The *pyramide* whose *Base* is the square of  $RX$  and *Altitude*  $RD$  is 87480.

The *parallelepipedon* whose *Base* is the square of  $ZX$  and *Altitude*  $XR$  is 437400.

Then  $729000 : 233280$  more  $87480$ , that is,  $320760 :: AX, 150 : BR, 48$ , more  $\frac{1}{2}$  of  $RX, 18$ ; that is  $66 :: 437400 : 192456$  equal to all the squares in the  $\frac{1}{4}$  of the *conoide*  $ZHRK$ ; but if that last number be multiplied by 4, the Product will be 769824 equal to all the squares in the *conoide*  $ZHRK$ , then  $14 : 11 :: 769824 : 533433 \frac{1}{2}$  the solidity of the *Hyperbolick conoide*  $ZHRK$ .

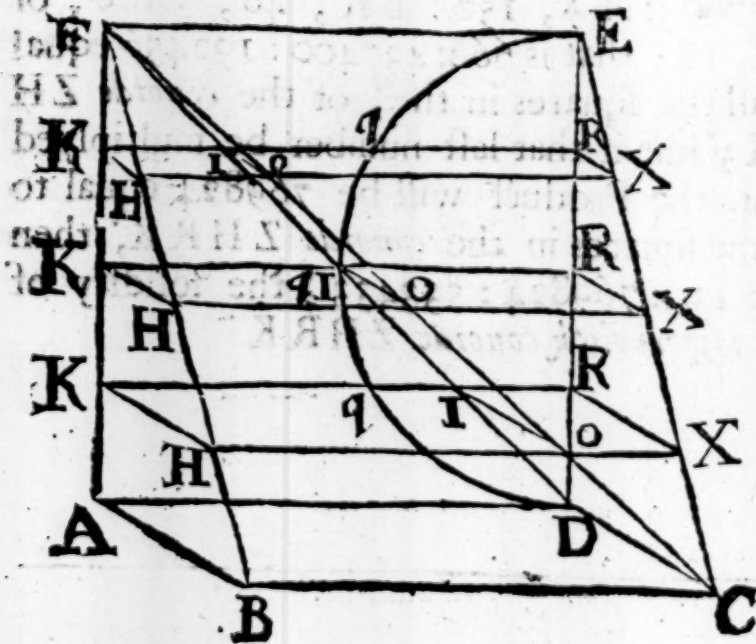
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**PROP.**

## PROPOSITION XV.

*To find the solidity of a whole Sphere, or a portion thereof.*

**L**et ADEF be a Plane and the Semi-circle DQIQE be in that Plane, Let the Planes ABCD and CDE be at Right Angle with the Plane ADEF; Let DC, AB and EF each of them be equal to DE;



Let

Let the Planes  $KHXR$  be parallel to the *Base* of the *prisme*; namely, parallel to  $AB$   $CD$ . The Rectangle  $DRE$  is equal to the square of  $RQ$ , by 35 3, that is, the Rectangle comprehended of  $IR$  and  $RX$ , for  $RX$  is equal to  $RE$ , and  $IR$  is equal to  $RD$ , therefore all the squares in  $\frac{1}{4}$  of the Sphere  $E$   $QIQD$  are equal to all the Rectangle in the *prisme*  $CDFE$ , the like in any part; Therefore the *prisme*  $RXHKFE$ , less the *pyramide*  $OIKHF$  shall be equal to all the squares in the portion of the Sphere  $QER$ . Then as 14 : 11, so the *prisme*  $XRIOFE$  to the solidity of the portion of the Sphere  $QER$ .

*In Numbers thus.*

Let  $AB$ ,  $AF$ ,  $FE$ ,  $AD$ ,  $DC$ ,  $BC$  each of these be equal to  $DE$ , and  $DE$  be equal to 204 : from  $E$  to the first  $R$  be 54, and from that first  $R$  to  $D$  be 150;  $KR$ , 204 multiplied by  $RX$ , 54; is  $XK$ , 11016; which being multiplied by  $\frac{1}{4}$   $RE$  27; the Product will be 297232, equal to the *prisme*  $KHXR$   $EF$ ;  $FK$ ,  $KH$ ,  $HO$ , and  $IK$ , every of them equal to 54; Therefore the *pyramide*  $KHO$   $IF$  will be 52488; then  $KHXR$   $EF$ , 297232; less  $KHO$   $IF$ , 52488 there will remain  $IOXR$   $EF$ , 244944; equal to all the squares in the portion  $QER$ ; that is, all the squares in the portion of  $\frac{1}{4}$  of a Sphere,

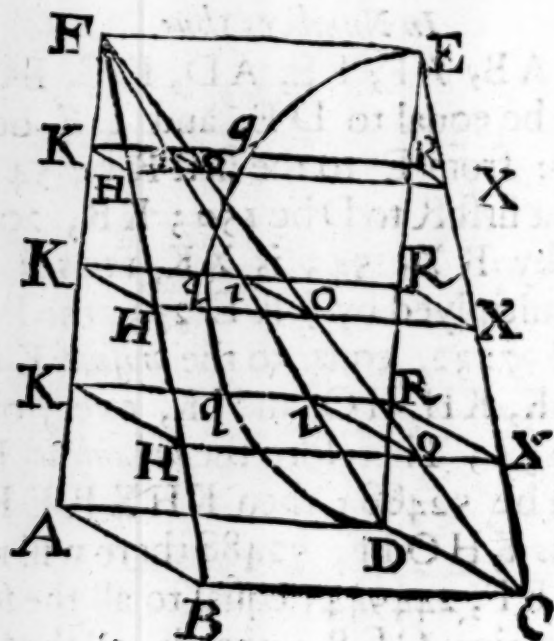
PROP.



## P R O P O S I T I O N X V I .

To find the solidity of a whole Spheroid, or a portion thereof.

**L** Et  $F A D E$  be a Plane, the *Ellipsis*  $E Q$   $D$  be in that Plane; Let  $A B$  and  $D C$  each of them be equal to  $D E$ , the Planes  $K R X H$  parallel to the Base of the *prisme*  $A B C D$ . Find the Line  $F E$  by the 10th. Prop. by the said 10th. Prop. the Rectangle  $I R E$



will

will be equal to the square of  $RQ$ ; but the Rectangle  $IRE$  is equal to the Rectangle  $IRX$ , because  $DC$  is equal to  $DE$ , therefore  $RX$  is equal to  $RE$ , therefore the Rectangle  $IRX$  is equal to the square of  $RQ$ ; therefore the *prisme*  $XHKREF$  less the *pyramide*  $KHOIF$  will be equal to all the squares in the portion  $QER$ .

*In Numbers Thus.*

Let  $DE$  the longest Diameter be 36, the Conjugate or shortest 18, therefore  $FE$  will be 12, for it is  $36 : 18 :: 18 : 12$ . Let from  $E$  to the first  $R$  be 9, then  $FE$ , 12, multiplied by  $RX$ , 9;  $XK$  will be 108; which being multiplied by  $\frac{1}{3}$  of  $ER$ , that is  $4\frac{1}{3}$  the Product will be 486, equal to  $KHXREF$ ; for the finding of  $KI$ , work thus,  $AF$ , 36; to  $AD$ , 12; so is  $FK$ , 9; to  $IK$ , 3; then  $IK$ , 3, multiplied by  $KH$  9; the Product is 27, that is,  $HI$ ; this last number being multiplied by  $\frac{1}{3}$  of  $KE$ , that is by 3; the *pyramide*  $KHOIF$  will be equal to 81, which being subtracted from the *prisme*  $KHXREF$  there will remain 405 the *prisme*  $IOXREF$  equal to all the squares in the portion of the *spheroid*  $QER$ .

PROP.

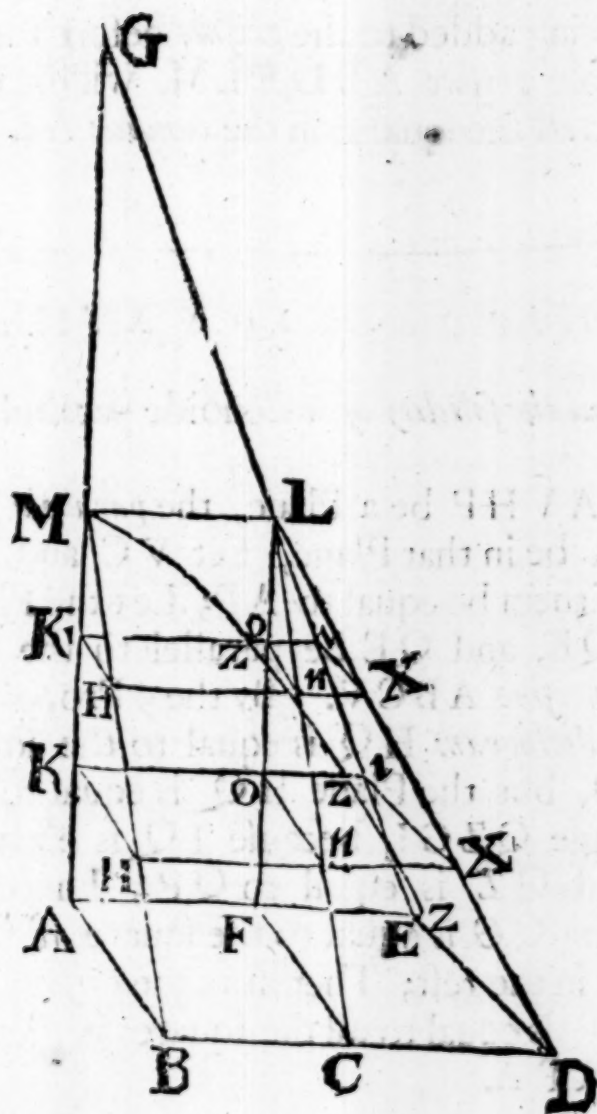
## P R O P O S I T I O N X V I.

To find the solidity of a hyperbolical Conoide.

**L** Et MAEL be a Plane, and the *hyperbola* be in that Plane; to that Plane let the Planes BAM, CEL and DEL be perpendicular; Let DE, CF and BA be each of them equal to AM; Let the Planes KIXH be parallel to the Plane ABDE the *Base* of the *prisme* ABDELM; find the Lines ML and MG by the 11 Proposition; the Rectangle IKM, is equal to the square of KZ, but the Rectangle IKM is equal to the Rectangle IKHX, because AB is equal to AM, therefore HK is equal to KM; Therefore the Rectangle IKHX, is equal to the square KZ; Therefore the *prisme* ABDELM, is equal to all the squares in the *conoide* AMZ.

In Numbers thus.

Let GM the Transverse Diameter be 18;  
 MA, 12; AE, 10; ML, 6; AB equal to  
 AM multiplied by AE, 16; the Product is  
 72 equal to ABCF, which being multiplied  
 by  $\frac{1}{3}$  AM, that is, 6, the Product is 432  
 equal to the *prisme* ABCFLM; FE, 4;  
 multi-



multiplied by  $FC$ , 12; the Product is 48, equal to  $FCDE$ , which being multiplied by  $\frac{1}{3}$  of  $FL$ , that is, 4; the Product is 192, equal to the *pyramide*  $FCDEL$ ; this *pyramide* being added to the *prisme* before found, the whole *prisme*  $ABDELM$ , will be 634, equal to all the squares in the *conoide*  $AZM$ .

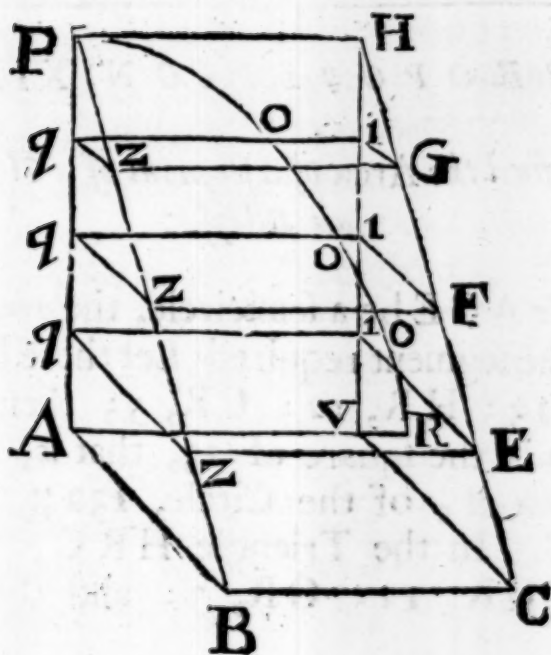
### PROPOSITION XVIII.

*To find the solidity of a Conoide parabola.*

**L** Et  $AVHP$  be a Plane, the *parabola*  $PO$   $RA$  be in that Plane, Let  $VC$  and  $AB$  each of them be equal to  $AP$ ; Let the Planes  $QG$ ,  $QF$ , and  $QE$  be parallel to the Base of the *prisme*  $ABCV$ . By the 9 Proposition the *parallelogram*  $HQ$  is equal to the square of  $QO$ , but the Plane  $HQ$  is equal to the Rectangle  $QZGI$ , because  $IQ$  is equal to  $HP$  and  $QZ$  is equal to  $QP$ ; Therefore the Plane  $QG$  is equal to the square of  $QO$ , the like in the rest; Therefore the *prisme*  $ABCVHP$  is equal to all the squares in the *conoide*  $RPA$ .

In





*In Numbers thus.*

Let  $AP$ , be 10;  $AV$ , 8; Therefore  $AC$  will be 80; which being multiplied by  $\frac{1}{2} AP$ , 10; that is, by 5, the Product will be 400, equal to the *prisme*  $ABCVHP$ ; that is, all the squares in  $\frac{1}{2}$  of a *parabolical conoid* whole Altitude is  $AP$ , 10; and semidiameter of the Base is  $AR$ .

In these 4 last Propositions it ought to be made, as 14:11; so are all the squares in the portion to the portion it self.

E

*Prop.*

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 PROPOSITION XIX.

*To find the Area of a segment of a Circle or an Ellepsis.*

**L** Et AGE be a semicircle, the area HGF the segment required; Let there be given AC, 13; HR, 12; CR, 5; then, as 14: 11, so is the square of 13, that is, 169, to the area of  $\frac{1}{4}$  of the Circle,  $132\frac{11}{14}$ , the area AGC. In the Triangle HRC, there is given HR, 12; CR, 5; and the Right



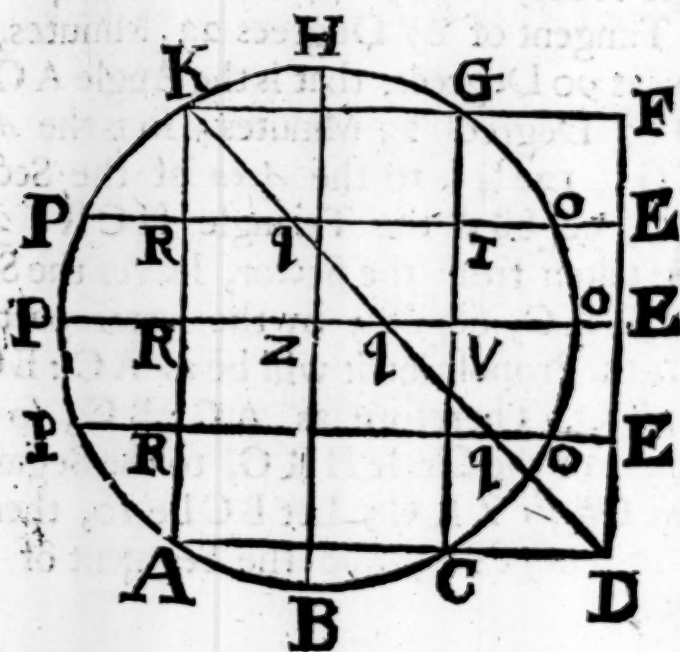
Angle

Angle  $GRH$ ; therefore there is given the Angle  $RCH$ ; thus,  $5 : 12 :: 100000 : 240000$ , the Tangent of 67 Degrees 23 Minutes, again, as 90 Degrees, that is the Angle  $ACG$ , is to 67 Degrees 23 Minutes; so is the Area  $ACG$ ,  $132\frac{11}{12}$ ; to the Area of the Sector  $HCG$ ,  $98\frac{88}{100}$ ; the Triangle  $HCR$ , 30; being taken from the Sector, leaves the Segment  $HRG$ ,  $68\frac{88}{100}$ ; by the latter part of the 10th. Proposition it will be as  $AC : BC :: IO : ZO$ , Therefore as  $AC : BC$ , so the Segment of the Circle  $HRG$ , to the Segment of the *Ellipsis*  $ZRG$ ; Let  $BC$  be 10, then as  $13 : 10 :: 68\frac{88}{100} : 52\frac{98}{100}$  the Segment of the *Ellipsis*.

# PROPOSITION XX.

To find the solidity of a Circular or Elliptick Spindle.

Let  $ABCOHP$  be a Circle,  $HB$  and  $LPVO$  be two Diameters at right Angle at  $Z$ ; Let  $AK$  and  $GC$  be parallel to  $HB$ , the Lines  $PO$  parallel to  $PVO$ ; Let  $GC$  equal to  $AK$  be the *Axis*,  $VO$  the Diameter of the Spindle. Suppose there be a Solid whose Base shall be the Segment  $KGO$   $CA$  and Altitude the Segment  $APK$ ; such



a Solid is equal to all the squares in  $\frac{1}{4}$  of a Sphere whose Diameter is  $AK$ ; for the Rectangle  $ORP$  is equal to the Rectangle  $ARK$ , 35, 3, and by the same 35, 3, the Rectangles  $ARK$  are equal to all the squares in  $\frac{1}{4}$  of a Sphere whose Diameter is  $AK$ .

If from the Solid whose Base is  $KGOCA$  and Altitude  $APK$ , there be taken a Solid, whose Base is  $KPA$  and Altitude  $KG$  there will remain a Solid whose Base and Altitude is equal to the Segment  $GOC$  or  $APK$ ; equal to the squares in the  $\frac{1}{4}$  of the Spindle.

By

By the 15 Proposition find all the squares in  $\frac{1}{4}$  of a Sphere whose Diameter is AK.

By the 19 Proposition find the *segment* APR which multiplied by KG makes a *parallelepipedon*, which being taken from  $\frac{1}{4}$  of the forementioned Sphere, leaves all the squares in  $\frac{1}{4}$  of the Spindle, this  $\frac{1}{4}$  being multiplied by 4 gives all the squares in the Spindle; Then, as 14: 11, so are all the squares in the Spindle, to all the Circles, that is the Spindle it self.

Further, as the whole so the parts are calculated; find the *segment* of a Sphere by the 15 Proposition, and the *segment* of the Circle KRP by the 19 Proposition, this *segment* KRP multiplied by KG, which taken from the corresponding part of the Sphere, leaves the corresponding part of the Spindle GOI.

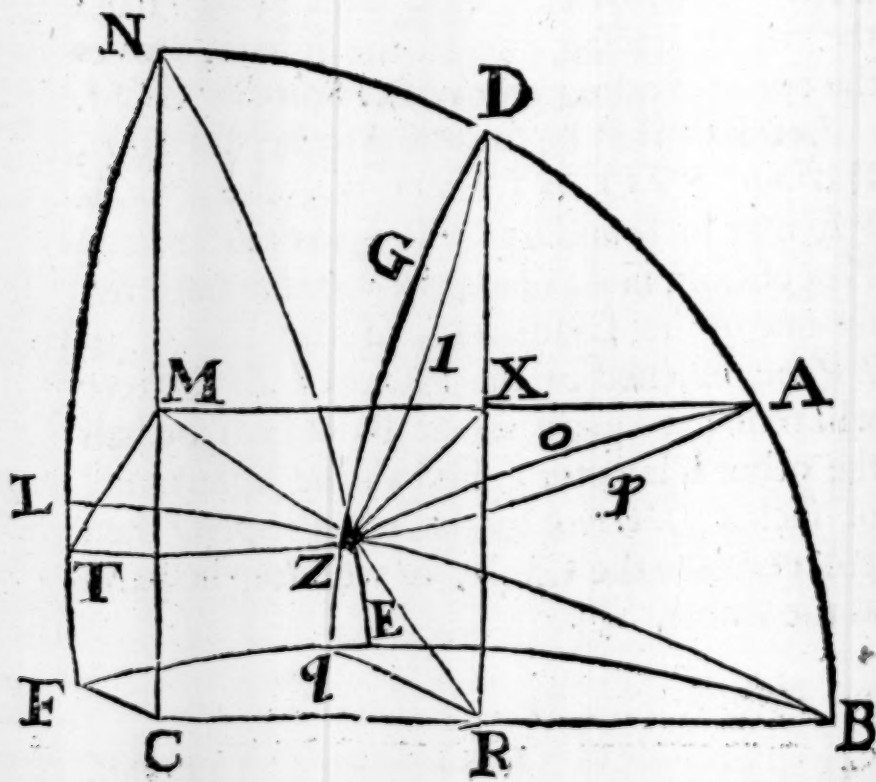
Let BOHP be supposed to be an *Ellipsis*, HB and PZO its Diameters; the *Area* KPA may be found by the last part of the 19th. Proposition: find  $\frac{1}{4}$  of a *Spheroid* that hath KA for one of its Diameters, if the square of ZP be lessened by the square of ZR, there will remain a square whose Root shall be half the other Diameter; find all the squares in  $\frac{1}{4}$  of such a *Spheroid* by the 16. Propos. then proceed as in the Circle, as well for the parts as the whole.



## P R O P O S I T I O N X X I.

To find the solidity of the second Sections in a Sphere or Spheroid.

**L** Et  $DABRCMNLFQ$  be  $\frac{1}{2}$  of a Sphere cut by two Planes, *viz.* the Plane  $RXDGZQ$  and  $MXAPZT$  being at right Angles, their Intersection  $XZ$ .  $MA$  Equal to  $MZ$ ;  $RD$  equal to  $RZ$ ;  $CN$  equal to  $CB$  the semidiameter of the Sphere:  $MX$  and  $XR$  being known, the rest may be found thus.



1.

The *Areas*  $ZXD$  and  $ZXA$  may be found by the 19th. Proposition.

2.

The Spherical *Superficie* of  $BDZQ$  is equal to a Surface whose *Base* is equal to the Line  $FLN$  and Altitude  $RB$ .

Or,

The Spherical *Superficie* of  $RDZQ$  is equal to the *Area* of a Circle whose Diameter is equal to a Line drawn from  $B$  to  $D$ .

The like for the Spherical *Superficie* of  $NAZT$ . *Archimedes* 36. 1. of the *Sphere* and *Cylinder*.

3.

Let  $RDGZQ$  and  $MAPZT$  be Quadrants of lesser Circles of the same Sphere,  $RD$  and  $MA$  their Semidiameters;  $CBZL$  and  $CNZE$  Quadrants of great Circles of the Sphere;  $AOZ$  and  $DIZ$  Arches of great Circles of that Sphere, the Arch  $NZ$  equal to the Arch  $NA$ , and the Arch  $BZ$  equal to the Arch  $BD$ ; the right lined Angle  $DRZ$  equal to the Spherical Angle  $DBZ$ , the Angle  $ZMA$  equal to the Angle  $ANZ$ .

4.

As 90 Degrees, is to the degrees and parts of a Degree in the Angle  $DBZ$ ; so is the Spherical *Superficie*  $BADGZQ$ , to the Spherical *Superficie* of the Triangle  $BADGZ$ .

E 4

In

5.

In the Triangle  $IZBDI$ ; there is given the Arches  $BD$  and  $BZ$ , and the Angle  $D$   $BZ$ ; Therefore there are given the Angles  $BDZ$  and  $BZD$ . The like in the Triangle  $OZNAO$  for the Angles  $NAZ$  and  $NZA$ ; by the third Case of Oblique angled Spherical Triangles.

6.

In the Triangle  $AOZID$ , there is given the Arch  $AD$  and the Angles  $ZAD$  and  $ZDA$ ; Therefore there is given the Angle  $AOZID$ , by the 8th. Case of Oblique angled Spherical Triangles.

7.

In the Triangle  $AOZID$  there are given the three Angles, Therefore the *Area* may be found, thus. As 180 Degrees, is to the excess of the three Angles over and above 180 Degrees; so is the *Area* of a great Circle of that Sphere to the *Area* of that Triangle, *Foster, Miscel.* page 21.

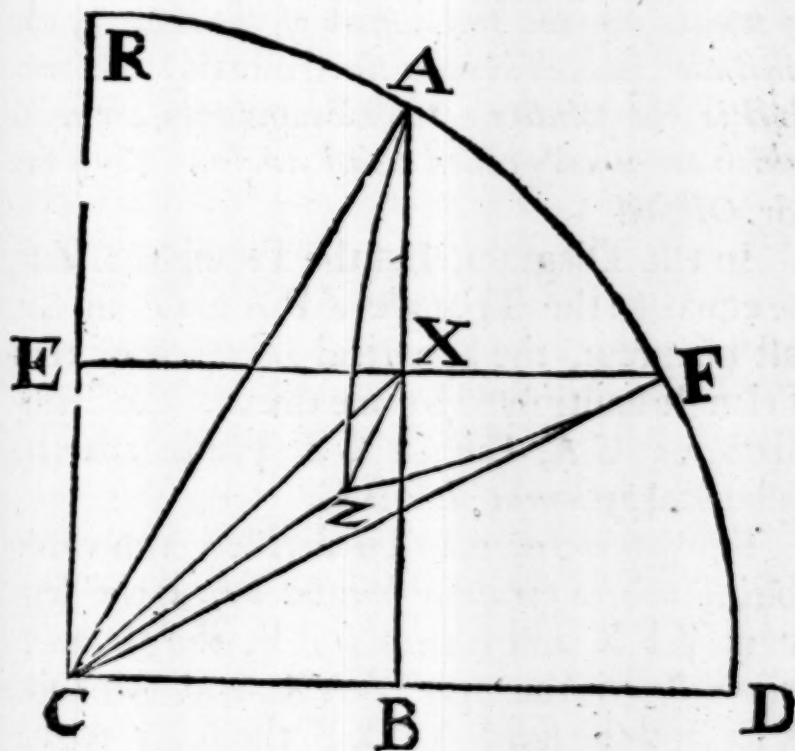
Or,

It may be found by that method which is delivered by that Learned Mathematician Mr. *John Leek*, in page, 116, of Mr. *Gibsons Syntaxis Mathematica*, Which is this. If the excess of the three Angles above 180 Degrees be multiplied by half the Diameter of the Sphere, the *Superficies* of any Spherical Triangle is thereby produced. By

By this Rule and by what Mr. *Gibson* delivered in that 116 page, I found this way to resolve this Proposition.

8.

By this last Rule we are to find the *Areas* of the Triangles B D I Z and N A O Z: the difference between the *Areas* of these, and those found by the 4. of this sheweth the *Areas* of the two Figures, *viz.* G Z I D and O Z P A, which added to the *Area* D I Z O A gives the *Superficie* of the mixt Lined Triangle A P Z G D.



9.

In the 9th. Chapter of the forementioned *Syntaxis Mathematica* there is given some Rules about some solids; but when it was time to treat of these second fragments, that Author breaks off thus; *There may be other parts of a Sphere besides those which are here called Fragments, (not to speak of those which are irregular and multiform) which are either Cones or Pyramides, whose Bases lie in the superficies of the Sphere, and their vertices at the Center, the solidity of one of these is found by multiplying the third part of the Base by the Altitude (which here is the femiaxis) the product is the solidity: these Fragments are those which are usually called Solid Angles.* Thus far Mr. Gibson.

In the Diagram, Let the Triangle  $AZF$ , be equal to the Triangle  $ZPADG$  in the last Diagram, the Spherical *superficie* of that Triangle multiplied by one third of the Semi-diameter  $CA$ ,  $CF$  or  $CZ$  produceth the Spherical *pyramide*  $ZFAC$ .

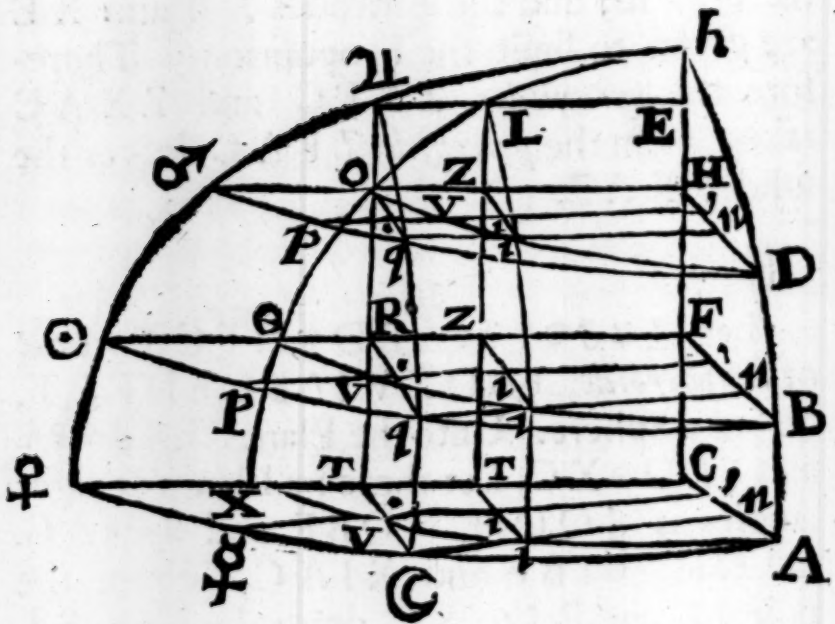
This *pyramide* may be divided into three solids, *viz.* a *pyramide* whose *Base* is the segment  $ZFX$  and *Altitude*  $ZB$ , the *pyramide* whose *Base* is the *Area*  $ZAX$  and *Altitude*  $XE$ ; and the solid  $AZXF$  the solid required. But the *pyramides* are given, because  
the



the *Areas*  $ZXF$  and  $ZXA$  may be found by the I of this, and the Altitudes  $XB$  and  $XE$  are given to limit the Proposition ; Therefore the *pyramides*  $ZXFC$  and  $ZXAC$  taken from the *pyramide*  $ZFAC$  leaves the solid  $F X A Z$ .

## 10.

Let  $h\gamma\delta\theta\varphi\zeta ABD h HFCT$  be  $\frac{1}{8}$  of a *Spherioide*,  $h\theta OXVIABD h HFCT$ , be  $\frac{1}{8}$  of a *Sphere*. Unto the Planes  $h\gamma\delta\theta\varphi C$  and  $hLO\theta XC$ , Let there be Planes at right Angles as  $\delta QDH$ ,  $\theta QBF$  and  $\varphi\zeta AC$ .  $OIDH$ ,  $\theta IBF$  and  $XIAC$ . From the points  $L$  parallel to  $\varphi C$  draw the Line  $\gamma LE$ , because the Line  $\gamma LE$  is parallel to  $\varphi C$ , Therefore the Line  $\gamma T$  is equal to  $LT$ , and the Line  $T\zeta$  is equal to  $TI$ , Therefore the Quadrants of the Circle  $T\gamma\zeta$  and  $TLI$  are equal. In these Planes, *viz.*  $\delta QDH$ ,  $\theta QBF$  and  $\varphi\zeta AC$ , Let there be Lines drawn, *viz.*  $P\text{ comma}$ , and  $QN$ , parallel to  $\delta H$ .  $P\text{ comma}$ , and  $QN$  parallel to  $\theta F$ .  $\varphi\text{ comma}$ , and  $\zeta N$  parallel to  $\varphi C$ . The points  $\delta PQD$ ,  $\theta PQD$  and  $\varphi\zeta A$  in those *Ellipses*, the points  $OVID$ ,  $\theta VIB$  and  $XVIA$  in those *Circles*.



$$\begin{aligned}
 \textcircled{\text{f}}\text{C} & : \text{XC} :: \textcircled{\text{m}}\text{H} : \text{OH, by the 10. Prop.} \\
 \textcircled{\text{f}}\text{C} & : \text{XC} :: \textcircled{\text{f}}\text{E} : \text{LE,} \\
 \textcircled{\text{m}}\text{H} & : \text{OH} :: \textcircled{\text{f}}\text{E} : \text{LE, 11. 5.} \\
 \textcircled{\text{m}}\text{H-OH} & : \text{OH} :: \textcircled{\text{f}}\text{E-LE} : \text{LE, 17. 5.} \\
 \textcircled{\text{m}}\text{O} & : \text{OH} :: \textcircled{\text{f}}\text{L} : \text{LE, 19. 5.}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{\text{f}}\text{F} & : \textcircled{\text{f}}\text{E} :: \textcircled{\text{f}}\text{F} : \text{LE, 10th. Prop.} \\
 \textcircled{\text{f}}\text{F-}\textcircled{\text{f}}\text{E} & : \textcircled{\text{f}}\text{E} :: \textcircled{\text{f}}\text{F-LE} : \text{LE, 17. 5.} \\
 \textcircled{\text{m}}\text{R} & : \textcircled{\text{f}}\text{E} :: \textcircled{\text{m}}\text{Z} : \text{LE, 19. 5.}
 \end{aligned}$$

$$\textcircled{\text{m}}\text{R} + \textcircled{\text{m}}\text{O} : \textcircled{\text{f}}\text{E} :: \textcircled{\text{m}}\text{Z} + \textcircled{\text{m}}\text{OZ} : \text{LE, 12. 5.}$$

$$\begin{aligned}
 \textcircled{\text{f}}\text{E} & : \text{LE} :: \textcircled{\text{f}}\text{C} : \text{XC,} \\
 \textcircled{\text{f}}\text{C} & : \text{XC} :: \textcircled{\text{m}}\text{R} + \textcircled{\text{m}}\text{O} : \textcircled{\text{m}}\text{Z} + \textcircled{\text{m}}\text{OZ, 11. 5.}
 \end{aligned}$$

$$\textcircled{\text{f}}\text{C} : \text{XC} :: \text{Area R}\textcircled{\text{f}}\text{C} : \text{Area Z}\textcircled{\text{m}}\text{OL,}$$

Again,

Again.

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$$\delta H : 4E :: (QN)OH:IN, \text{ 10. Prop.}$$

$$\delta H-QN:QN::OH-IN(ZH)IN, \text{ 17. 5.}$$

$$\delta O : OH::OZ : ZH, \text{ 19. 5.}$$


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$$P, : QN::V, : IN,$$

$$P,-QN:QN::V,-IN:IN, \text{ 17. 5.}$$

$$P, : QN::V, : IN, \text{ 19. 5.}$$


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$$\delta O+P, : QN::(4E)OZ+VI:IN; (LE,) \text{ 12.5.}$$

$$\delta O+P, : 4E::OZ+VI:LE,$$

$$\text{♀}C : XC::4E : LE,$$

$$\text{♀}C : XC::\delta O+P, : OZ+VI, \text{ that is,}$$

$$\text{♀}C:XC::\text{Area } OQP\delta : \text{Area } ZIVO, \text{ the Plane.}$$

○ P Q B F or any other drawn parallel to these, shall have the same qualifications; which makes us conclude that the whole Section I L T X shall be to the whole Section 4 T ♀ as X C is to ♀ C, and as X C to ♀ C so the segment I L Z O to the segment Q 4 O δ, the second Section in the *Spheroid*.

---

P R O P.

---

 P R O P O S I T I O N XXII.

**I**F four Numbers be in proportion, that is, as the first is to the second; so is the third, to the fourth. It will always be, as the first, is to a Geometrical mean proportion between the first and second; so is the third, to a Geometrical mean proportion between the third and fourth.

Let it be,

$A : B :: C : D$ , multiply the two first terms by  $A$ , and it will be, as  $AA : AB :: C : D$ , 17. 7. If the two last terms be multiplied by  $C$ , it will be  $AA : AB :: CC : CD$ , then, as  $A : \sqrt{AB} :: C : \sqrt{CD}$ , 22. 6.

*In Numbers thus.*

$$4 : 9 :: 16 : 36.$$


---

$$16 : 36 :: 16 : 36.$$


---

$$16 : 36 :: 256 : 576.$$


---

$$4 : 6 :: 16 : 24.$$


---

P R O P.

---

 PROPOSITION. XIII.

To find the Relation of one Hyperbola  
to another.

**L** Et FLBC be a *semihyperbola*, its Transverse Diameter FH, its intercepted Axis FC. Let GMB C be another *semihyperbola*, its Transverse Diameter K G, its intercepted Axis G C.

Let it be made, as,

1.

|       |   |    |    |          |   |          |        |
|-------|---|----|----|----------|---|----------|--------|
| KG    | : | GC | :: | HF       | : | FC,      |        |
| KG+GC | : | GC | :: | HF+FC    | : | FC,      | 18. 5. |
| KC    | : | GC | :: | HC       | : | FC       |        |
| KC    | : | HC | :: | GC       | : | FC,      | 16. 5. |
| KC    | : | HC | :: | GC       | : | FC,      | 17. 7. |
|       |   |    |    | <u>2</u> |   | <u>2</u> |        |

|     |   |     |    |     |   |                  |         |
|-----|---|-----|----|-----|---|------------------|---------|
| KC  | : | HC  | :: | GF  | : | FE               |         |
| KF  | : | HE  | :: | GF  | : | FE,              | 19. 5.  |
| KC  | : | HC  | :: | KF  | : | HE,              | 11. 5.  |
| GC  | : | FC  | :: | GF  | : | FE,              | 11. 5.  |
| KCG | : | HCF | :: | KFG | : | HEF,             | 23. 6.  |
| KCG | : | KFG | :: | HCF | : | HEF,             | 16. 5.  |
| HCF | : | HEF | :: | CBC | : | ELE,             | 11 Pro. |
| KCG | : | KFG | :: | CBC | : | FMF=ELE,         |         |
|     |   |     |    |     |   | 11. 5. 11. Prop. |         |
|     |   |     |    |     |   | Therefore        |         |



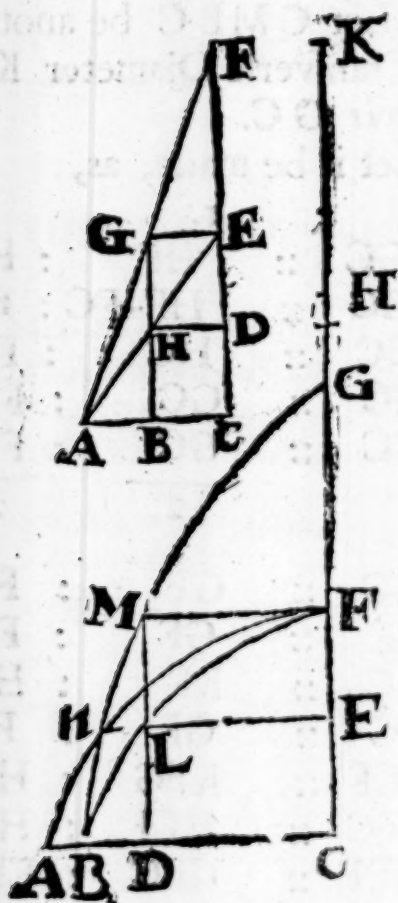
Therefore FM is equal to the Line LE, and for as much as GC is divided in F, in the same Ratio as FC is in E, Therefore,

$$\mathbf{GG} : \mathbf{FC} :: \mathbf{GF} : \mathbf{FE}, \quad 17.7.$$

**GC : FC :: FC : EC, 19. 5.**

**GC : FC :: MD : LD,**

$$GC : FC :: \text{area CBG} : \text{area CBF}_{12.5}$$



**From**

2.

From the *Vertex* of the *Hyperbola*  $\mathbf{CBF}$ ,  
Let there be a *Semihyperbola* as  $\mathbf{CAF}$ ;  $\mathbf{FH}$   
its *Transverse Diameter*.

Then as,

$$\begin{array}{llll} \mathbf{HCF} : \mathbf{HEF} :: \mathbf{CBC} & : \mathbf{ELE}, & 11. \text{ Prop.} \\ \mathbf{HCF} : \mathbf{HEF} :: \mathbf{CAC} & : \mathbf{ENE}, & 11. \text{ Prop.} \\ \mathbf{CBC} : \mathbf{CAC} :: \mathbf{ELE} & : \mathbf{ENE}, & 11. 5. \\ \mathbf{CB} : \mathbf{CA} :: \mathbf{EL} & : \mathbf{EN} & 22. 6. \\ \mathbf{CB} : \mathbf{CA} :: \text{area } \mathbf{CBF} : \text{area } \mathbf{CAF}, & 12. 5. \end{array}$$

3.

Let  $\mathbf{CVQ}$  be  $\frac{1}{2}$  of an *Ellipsis*, its *Transverse Diameter*  $\mathbf{FV}$ , its *semiconjugate*  $\mathbf{CQ}$ ,  
Let  $\mathbf{CGQ}$  be  $\frac{1}{2}$  of another *Ellipsis* its *Transverse Diameter*  $\mathbf{EG}$ , its *conjugate*  $\mathbf{CQ}$ .

Let it be made, as,

$$\begin{array}{llll} \mathbf{FC} : \mathbf{EC} & :: \mathbf{CV} & : \mathbf{CG}, \\ \mathbf{FC} : \mathbf{EC} & :: \frac{\mathbf{CV}}{2} & : \frac{\mathbf{CG}}{2}, & 17. 7. \\ \mathbf{FC} : \mathbf{EC} & :: \mathbf{CG} & : \mathbf{CD}, & 17. 7. \\ \mathbf{FC} : \mathbf{CG} & :: \mathbf{EC} & : \mathbf{CD}, & 16. 5. \\ \mathbf{FC} : \mathbf{FC} + \mathbf{CG} & :: \mathbf{EC} & : \mathbf{EC} + \mathbf{CD}, & 18. 5. \\ \mathbf{FC} : \mathbf{FG} & :: \mathbf{EC} & : \mathbf{ED}, & 18. 5. \\ \mathbf{CV} : \mathbf{GV} & :: \mathbf{CG} & : \mathbf{DG}, & 17. 7. \\ \mathbf{FCV} : \mathbf{FGV} & :: \mathbf{ECG} & : \mathbf{EDG}, & 23. 6. \\ \mathbf{FCV} : \mathbf{FGV} & :: \mathbf{CQC} & : \mathbf{GLG}, & 10. \text{ Prop.} \\ \mathbf{ECG} : \mathbf{EDG} & :: \mathbf{CQC} & : \mathbf{DHD} = \mathbf{GLG}, & \\ & & & 10. \text{ Prop.} \\ & & & \text{Therefore} \end{array}$$

F

Therefore DH and GL are equal, Because the Line CV is divided in G in the same ratio as CG is in D.

Therefore, as,

$$\begin{array}{llll} \text{CV} : \text{VG} :: \text{CG} & : \text{GD}, & & \\ \text{CV} : \text{CG} :: \text{CG} & : \text{DC}, & 19.5. & \\ \text{CV} : \text{CG} :: \text{IL} & : \text{IH}, & & \\ \text{CV} : \text{CG} :: \text{area CQY} : \text{area CQG}, & & & \end{array}$$

4.

From the vertex of the Quarter of the Ellipsis CGQ; Let there be  $\frac{1}{2}$  of another Ellipsis, as CGR, its Transverse Diameter EG the semiconjugate CR,

Then, as,

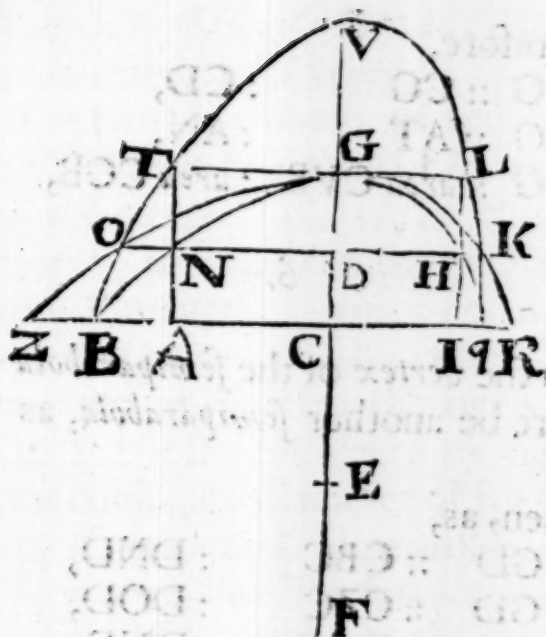
$$\begin{array}{llll} \text{ECG} : \text{EDG} :: \text{CQC} & : \text{DHD}, & 10. \text{Pr.} & \\ \text{ECG} : \text{EDG} :: \text{CRC} & : \text{DKD}, & 10. \text{Pr.} & \\ \text{CQC} : \text{DHD} :: \text{CRC} & : \text{DKD}, & 11.5. & \\ \text{CQ} : \text{CR} :: \text{DH} & : \text{DK}, & 22.6. & \\ \text{CQ} : \text{CR} :: \text{area CDQ} : \text{area CGR}, & 12.5. & & \end{array}$$

5.

Let CGB be half a parabola, its Diameter CG, its Ordinate CB. Let CVB be half of another parabola, its Diameter CV, its Ordinate CB.

Let

Therefore  $GT$  is equal to  $DN$ , because  
the line  $CV$  is divided in  $G$  in the same  
ratio as  $CG$  is to  $D$ .



Let it be,

$$CV : CG :: CV : CG,$$

$$CV : CG :: \frac{CV}{2} : \frac{CG}{2},$$

$$CV : CG :: GV : GD,$$

$$CG : GD :: CBC : DND,$$

$$CV : GV :: CBC : GTG = DND,$$

9. Prop.

9. Prop.

F 2

Therefore,

Therefore GT is equal to DN; because the Line CV is divided in G in the same *ratio* as CG is in D.

Therefore,

$$\begin{array}{llll} \text{CV} : \text{CG} :: \text{CG} & : \text{CD}, \\ \text{CV} : \text{CG} :: \text{AT} & : \text{AN}, \\ \text{CV} : \text{CG} :: \text{area CVB} & : \text{area CGB}, \quad 12. 5. \end{array}$$

6.

From the *vertex* of the *semiparabola* CGB; Let there be another *semiparabola*, as CGZ,

Then, as,

$$\begin{array}{llll} \text{GC} : \text{GD} :: \text{CBC} & : \text{DND}, & 9. \text{ Pr.} \\ \text{GC} : \text{GD} :: \text{CZC} & : \text{DOD}, & 9. \text{ Pr.} \\ \text{CZC} : \text{CBC} :: \text{DOD} & : \text{DND}, & 11. 5. \\ \text{ZC} : \text{BC} :: \text{OD} & : \text{ND}, & 22. 6. \\ \text{ZC} : \text{BC} :: \text{area ZGC} & : \text{area BGC}, & 12. 5. \end{array}$$

7.

Hence,

It follows that the *areas* of *hyperbolas*, *Ellipses* and *parabolas* may be increased or decreased in any proportion assigned; either according to their Transverse diameters or Ordinates: or both together.

It



It follows also,

That the *areas* of *hyperbolas*, *ellipses* and *parabolas* of the same *Bases* are as their *Altitudes*

And also,

That the *areas* of *hyperbolas*, *ellipses* and *parabolas* of the same *Altitudes* are as their *Bases*.

Further it follows.

If there be two *hyperbolas*, namely, A and B; if the Transverse Diameter of A, be to the Transverse Diameter of B; as the conjugate Diameter of A, is to the conjugate Diameter of B; if the Transverse Diameter of A, is to the Transverse Diameter of B; as the intercepted diameter of A, is to the intercepted diameter of B: if the conjugate diameter of A, is to the conjugate diameter of B; as the Ordinate of A, is to the Ordinate of B. Then such *hyperbolas* are called like *hyperbolas*.

And the *area* of A, will be to the *area* of B; as the Rectangled Figure of the Transverse and conjugate diameters of A, to the Rectangled Figure of the Transverse and conjugate diameters of B.

Or,

The *area* of A, will be to the *area* of B; as the Rectangled Figure of the intercepted diameter, and Ordinate of A, is to the Rectangled Figure of the intercepted diameter and Ordinate of B.

(70)

8.

Here note,

In the *hyperbolas* CFB, CFA and CGB, the Lines CB, EL; and CA, EN; and CB, FM, are called Ordinates.

In the *Ellipses*.

CGQ, CGR and CVQ; the Lines DH, and DK and GL are Ordinates; the Lines CQ and CR conjugate semidiameters.

In the *parabolas*.

CGB, CGZ and CVB; the Lines CB, DN; and CZ, DO; and CB, GT are called Ordinates.

9.

In the *hyperbola* CFB; CF, EF, CB and EL being given, to find the Transverse diameter FH.

Let  $FE = A \cdot FC = B \cdot CB = D \cdot EL = E \cdot Z = HF$ .  
Therefore  $HE = Z + A$ .  $HC = Z + B$ .

Therefore,

$DD : EE :: ZB + BB : ZA + AA$ , 11. Prop.

---

$ZADD + AADD = ZBEE + BBEE$ , 16. 6.

---

$-ZBEE$

$-AADD$ .

---

*In Words thus.*

The difference between the square of B in the square of E, and the square of A in the square of D; being divided by the difference between A in the square of D and B in the square of E; the Quotient is the value of Z.

10.

In the *Ellipsis* C Q G; C Q, D H, and CD being given, to find the Transverse diameter E G.

Let CQ=A. DH=B. CD=D, GC=Z.  
Therefore  $Z+D=ED$ .  $Z-D=DG$ .

$ZZ : Z^{\dagger} \text{ in } Z-D :: AA : BB$ , 10. Prop.

---


$$ZZBB = ZZAA - DDAA. \quad 16. 6.$$


---

$$\begin{array}{r} DDAA = ZZAA \\ - ZZBB \end{array}$$

*In Words thus.*

As the square Root, of the difference between the square of A and the square of B, is to A, so is D, to Z, the *semitransverse* diameter. 22. 6.

## II.

In the *parabola*  $CGB$ ;  $CB$ ,  $CD$  and  $DN$  being known, to find the diameter  $DG$ .

Let  $CB=A$ .  $DN=B$ .  $CD=C$ .  $DG=Z$ . Then  
 $C+Z=CG$ .  $AA : BB :: C+Z : Z$ , 9. Prop.

---


$$AAZ=BBC+BBZ$$

16. 6.

---


$$-BBZ$$


---

*In Words thus.*

As the difference betwixt the square of  $A$  and  $B$ , is to the square of  $B$ ; so is  $C$ , to  $Z$ ,

Or thus.

$$\begin{array}{lcl} AA : BB & :: C+Z : Z, & 9. \text{ Prop.} \\ AA-BB : BB & :: C+Z-Z : Z, & 17. 5. \end{array}$$

## I2.

Yet further, it may be made manifest, that by the points of  $B$  and  $F$  there may be infinite hyperbolick lines pass : the *area* of the least, shall not be so little as half the *parallelogram*, whose *base* is  $BC$ , and *Altitude*  $CE$  : nor the *area* of the greatest, so great, as three fourths of the said *parallelogram*.

PROP.

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 PROPOSITION XXIV.

*Of all manner of cylindrick hoofs.*

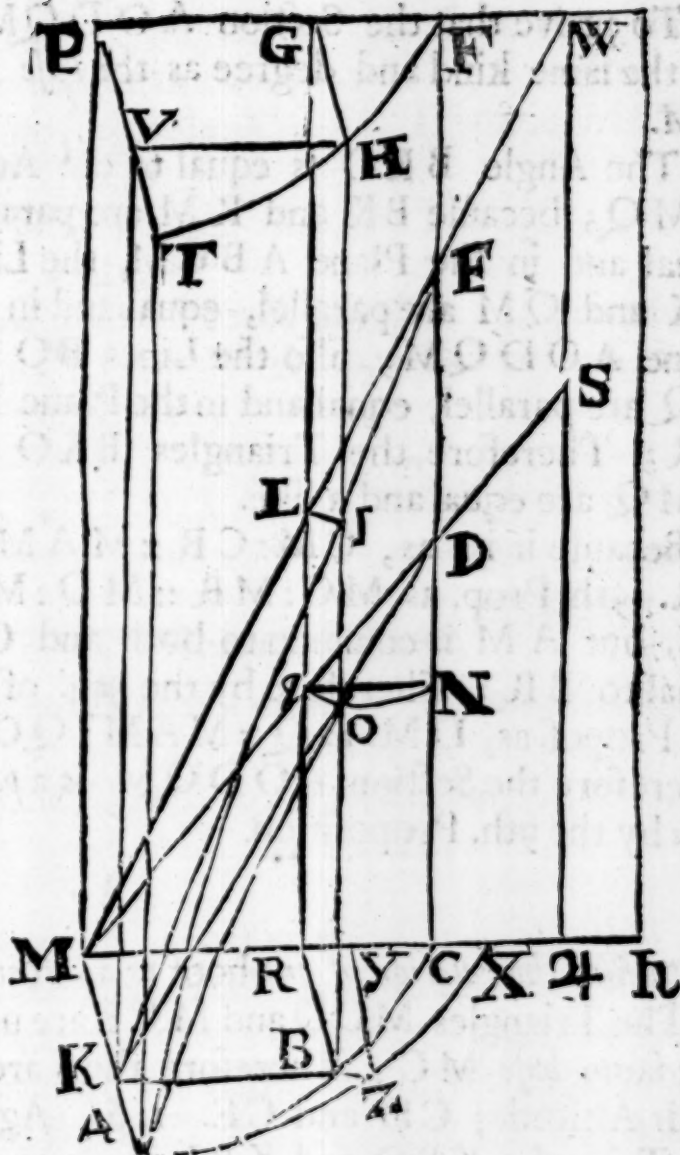
I.

**L** Et  $MACFTP$  be half a *parabolick* Cylinder, the base  $MAC$  parallel, equal and a like to the base  $FTP$ . Let  $C$  and  $F$  be the vertices of the parabolas  $ACM$  and  $TFP$ ,  $CM$  and  $FP$  their diameters. Let this cylinder be cut by the Plane  $HVKB$  parallel to the Plane  $FPMC$ , the Line  $HB$  in the *superficie* of the cylinder. Let it be cut by another Plane, as  $HGRB$  parallel to the Plane  $PTAM$ , the line  $HB$  in the *superficie* of the said cylinder. Let it be cut by the Plane  $OADQM$ , the line  $AOD$  in the *superficie* of the cylinder, and the line  $DQM$  in the Plane  $CFPM$ ; the Ordinate  $AM$  the common Section of the Planes  $ABCM$  and  $OADQM$ . Further, Let it be cut by the Plane  $AIELM$ , the line  $AIE$  in the *superficie* of the cylinder and the line  $ELM$  in the Plane  $CFPM$ . From  $K$ , to  $O$  and  $I$  let lines be drawn, and also from  $M$  to  $D$  and  $E$ : from the intersection of  $GR$  and  $ME$ , that is from  $L$ , to the intersection of the lines  $HB$  and  $AE$ , that is to  $I$ , let there be a line drawn, as  $LI$ ; and



and likewise the line  $QO$ ; then the lines  $LI$  and  $QO$  will be equal and parallel to the line  $RB$ . Because the Planes  $AODQM$  and  $AIELM$ , cut the Plane  $GFP M$  at right Angles, and the points  $B, O, I, H$ , are in the *superficie* of the *cylinder* and in the line  $BH$ .

Betwixt the Planes  $ACM$  and  $AODQM$  there is a solid made, as  $ABCMQDOA$ ; also betwixt the said *base*  $ABCM$  and the Plane  $AIELM$  there is another solid made as  $ABCMLEIA$ : such solids are called *cylindrick hoofs*, and they take their particular names from such *cylinders* as they are part of; *viz.* if the *base*  $ABCM$  be half, or a quarter of a circle, it may be called a circular *cylindrick hoof*; If half or a quarter of an *ellipsis*, then an *elliptick cylindrick hoof*. If the *base* be half, or a whole *parabola*, then a *parabolick cylindrick hoof*. If half, or a whole *hyperbola*, then a *hyperbolick cylindrick hoof*; the like for any other.



To

2.

To prove that the Section  $AODQM$  is of the same kind and degree as the *base*  $ABCM$ .

The Angle  $BKO$  is equal to the Angle  $RMQ$ ; because  $BK$  and  $RM$  are parallel, equal and in the Plane  $ABCM$ , the Lines  $OK$  and  $QM$  are parallel, equal and in the Plane  $AODQM$ , also the Lines  $BO$  and  $RQ$  are parallel, equal and in the Plane  $BHGR$ ; Therefore the Triangles  $BKO$  and  $RMQ$  are equal and alike.

Because it is, as,  $CM:CR::MAM:RBR$ . 9th. Prop. as  $MC:MR::MD:MQ$ . 4.6. but  $AM$  is common to both and  $OQ$  equal to  $BR$ ; Therefore by the 5th. of the 23. Propos. as,  $DM:DQ::MAM:QOQ$ . Therefore the Section  $AODQM$  is a *parabola* by the 9th. Proposition.

3.

*To find the relation of one hoof to another.*

The Triangles  $MCD$  and  $MCE$  are upon the same *base*  $MC$ , Therefore they are as their Altitudes  $CD$  and  $CE$ . 1.6. Again, the Triangles  $KBO$  and  $KBI$  are upon the same *base*  $KB$ , Therefore they are as their Altitudes  $BO$  and  $BI$ , 1.6.

Further,

Further, the Triangles  $KBO$  and  $MCD$  are alike, and also the Triangles  $KBI$  and  $MCE$  are alike, therefore their sides are in proportion. 4. 6.

The Triangle  $MCD$  : Triangle  $MCE$  ::  
 $CD$  :  $CE$ . 1. 6.

The Triangle  $KBO$  : Triangle  $KBI$  ::  
 $BO$  :  $BI$ . 1. 6.

But  $BO$  :  $BI$  ::  $CD$  :  $CE$ . 4. 6.

The Triangle  $KBO$  : Triangle  $KBI$  ::  
 $CD$  :  $CE$ . 11. 5.

---

$KBO + MCD$  :  $KBI + MCE$  ::  $CD$  :  $CE$ . 12. 5.

---

Hence it follows,

That the solidities of *hoofs* upon the same base are as their Altitudes, that is, the *hoof*  $ABCMQDOA$ , is to the *hoof*  $ABCMLEIA$ ; as  $CD$ , is to  $CE$ . †

Further it follows.

Because it is  $BO$  :  $BI$  ::  $CD$  :  $CE$ , Therefore the *superficies* of *hoofs*, upon the same base are as their Altitudes, that is, the *superficie*  $CBAOD$ , is to the *superficie*  $CBAIE$ ; as  $CD$ , is to  $CE$ .

*To find the solidity of cylindrick hoofs.*

The Triangles  $MCD$  and  $KBO$  are alike, Therefore as, as  $KB : BO :: MC : CD$ , 4. 6. and  $KB : \frac{BO}{2} :: MC : \frac{CD}{2}$  by the converse of 17. 7. then as,  $KB$ , to a Geometrical mean proportion between  $KB$  and half  $BO$ , so is  $MC$ , to a Geometrical mean proportion between  $MC$  and half  $CD$ , by the 22. Proposition. Let  $MX$  be made equal to the last term in the last Proportion, viz. the Geometrical mean proportion between  $MC$  and half  $CD$ . Then will  $KZ$  be equal to a Geometrical mean proportion between  $KB$  and half  $BO$ , and  $AZXM$  will be a *semi-parabola* by the 5th. of the 23. Proposition; that is, as  $CM : XM :: BK : ZK$ . Further, the *area* of the Triangle  $MCD$  is equal to the Product of  $MC$  in half  $CD$ , that is, a Square whose side is a Geometrical mean proportion between  $MC$  and half  $CD$ , that is, equal to the Square of  $MX$ . Also the *area* of the Triangle  $KBO$  is equal to the Product of  $KB$  in half  $BO$ , that is, equal to a square whose side is a Geometrical mean proportion between  $KB$  and half  $BO$ ; that is equal to the Square of  $KZ$ .

Hence



5.

Hence it follows.

That the solidity of the *hoof* MQDOA BC is equal to all the squares in one eighth of a *parabolick Spindle* whose *semiaxis* is AM and *semidiameter* is MX; but all the squares in a *parabolick Spindle* is to a *parallelepipedon* of the same *Base* and *Altitude*; as 8, is to 15. *Bonav. Caval. Exerc. quer. page 282.*

6.

It further follows.

As all the squares in the whole solid AZ XM are equal to the whole *hoof* MQDO AC, so are their parts equal, if the *axis* be cut by a Plane at right Angle.

*Example.*

The Plane BKO cuts the *axis* AM in K at right Angle, that is, the Plane KBO is parallel to the Plane MCD, the part of the *Spindle* AKZ, is equal to the part of the *hoof* AKOB.

7.

If the *hoof* ABCMQDOA be cut by a Plane QON parallel to the *Base* ABCM, the part QOND may be found thus.

First, take away the part KABO equal to the part of the *Spindle* KAZ: then take away  
the

the *prism*  $e K B R M Q O$ ; Lastly, take away the *parabolick Semicylinder*  $R B C N O Q$ , there will remain the little *hoof*  $Q O N D$ .

## 8.

If  $A C M$  be a *semihyperbola* its Transverse diameter  $C \mu$ , the *hyperbolick cylinder* may be  $M A B C F H T P$ : this *cylinder* being cut by Planes according to the *First* of this Proposition, the Sections  $A O D Q M$  and  $A I F L M$  will be *semihyperbolas*,  $D S$  and  $E W$  their Transverse diameters by the *First* of the 23d. Proposition. If between  $M \mu$  and half  $\mu S$  there be a Geometrical mean proportion found, suppose it  $M h$ , and if between  $M C$  and half  $D C$  there be taken a Geometrical mean proportion, suppose it  $M X$ : And also if a Geometrical mean proportion be taken between  $K B$  and half  $B O$ , suppose it  $K Z$ ; the Plane  $A Z X M$  will be the *area* of a *hyperbola* its Transverse diameter  $X h$ ; by the 22d. Proposition and by the *First* of the 23d. Proposition all the Squares in one Fourth of the *hyperbolick Spindle*  $A Z X M$  taking  $A M$  for its *axis*, will be equal to the *hyperbolick hoof*  $A B C M Q D O A$ , by the 4th. and 5th. of this Proposition. The relations of the solidities and *superficies*, by the Third of this.

If

9.

If  $A B C M$  be a quarter of a Circle, the circular *cylinder* will be  $A B C M T H F P$ ; this *cylinder* being cut by Planes, according to the *First* of this, the Sections  $A O D Q M$  and  $A I E L M$  will be quarters of *Ellipses* by the *Third* of the 23d. Proposition. If a Geometrical mean proportion be taken between  $M C$  and half  $C D$ , suppose it  $M X$ , and also a Geometrical mean proportion be taken between  $K B$  and half  $B O$ , suppose it  $K Z$ . The area  $A Z X M$  will be one quarter of an *Ellipsis* by the 22d. Proposition, and by the *Third* of the 23d. Proposition.

All the squares in one fourth of the *semi-spheroid*, taking  $A M$  for its *Semixis*, and  $M X$  for its *semidiameter*, will be equal to the circular *cylindrick hoof*  $A O D Q M C B A$ , by the *Fourth* and *Fifth* of this Proposition. As the whole, so the parts, according to the *Sixth* and *Seventh* of this Proposition. How to find all the squares in any Sphere or *spheroid* is taught Proposition the 15th. and 16th.

Here Note,

If the Altitude of the *hoof* be equal to four diameters of the *base*, that *hoof* will be equal to all the squares in a Sphere adscribed in that *cylinder*.

G

If

If the Altitude be less than Four diameters of the *base*, that *hoof* will be equal to all the squares in a *spheroid* whose longest diameter shall be the *axis*.

But if the Altitude be greater than Four diameters of the *base*, then that *hoof* will be equal to all the squares in a *spheroid* whose shortest diameter shall be the *axis*.

## 10.

If  $ABCM$  be a quarter of an *Ellipsis*, the *Elliptick cylinder* will be  $ABCMTHFP$ ; this *cylinder* being cut by *Planes*, according to the First of this, the Sections  $AODQM$  and  $AIELM$  will be quarters of *Ellipses* by the 3d. of the 23d. Proposition.

If Geometrical means be taken between  $MC$  and half  $CD$ , and also between  $KB$  and half  $BO$ , suppose them to be  $MX$  and  $KZ$ , Then will  $AZXM$  be a quarter of an *Ellipsis* by the 22d. Proposition, and by the 3d. of the 23d. Prop. all the squares in one Fourth of a *semispheroid* taking  $AM$  for its *semiaxis* and  $MX$  for its *semidiameter*, will be equal to the *Elliptick cylindrick hoof*  $AODQMCBA$  by the Fourth and Fifth of this Proposition. The parts  $KABOBC$   $RQON$  and  $QOND$  are found as in the Sixth and Seventh of this.



*A Table of Squares and Cubes.*

| Roots. | Squares. | Cubes. | Roots. | Squares. | Cubes. |
|--------|----------|--------|--------|----------|--------|
| 1      | 1        | 1      | 25     | 625      | 15625  |
| 2      | 4        | 8      | 26     | 676      | 17576  |
| 3      | 9        | 27     | 27     | 729      | 19683  |
| 4      | 16       | 64     | 28     | 784      | 21952  |
| 5      | 25       | 125    | 29     | 841      | 24389  |
| 6      | 36       | 216    | 30     | 900      | 27000  |
| 7      | 49       | 343    | 31     | 961      | 29791  |
| 8      | 64       | 512    | 32     | 1024     | 32768  |
| 9      | 81       | 729    | 33     | 1089     | 35937  |
| 10     | 100      | 1000   | 34     | 1156     | 39304  |
| 11     | 121      | 1331   | 35     | 1225     | 42875  |
| 12     | 144      | 1728   | 36     | 1296     | 46656  |
| 13     | 169      | 2197   | 37     | 1369     | 50653  |
| 14     | 196      | 2744   | 38     | 1444     | 54872  |
| 15     | 225      | 3375   | 39     | 1521     | 59319  |
| 16     | 256      | 4096   | 40     | 1600     | 64000  |
| 17     | 289      | 4913   | 41     | 1681     | 68921  |
| 18     | 324      | 5832   | 42     | 1764     | 74088  |
| 19     | 361      | 6859   | 43     | 1849     | 79507  |
| 20     | 400      | 8000   | 44     | 1936     | 85184  |
| 21     | 441      | 9261   | 45     | 2025     | 91125  |
| 22     | 484      | 10648  | 46     | 2116     | 97336  |
| 23     | 529      | 12167  | 47     | 2209     | 103823 |
| 24     | 576      | 13824  | 48     | 2304     | 110592 |



*A Table of Squares and Cubes.*

| Roots. | Squares. | Cubes. | Roots. | Squares. | Cubes. |
|--------|----------|--------|--------|----------|--------|
| 49     | 2401     | 117649 | 73     | 5329     | 389017 |
| 50     | 2500     | 125000 | 74     | 5476     | 405224 |
| 51     | 2601     | 132651 | 75     | 5625     | 421875 |
| 52     | 2704     | 140608 | 76     | 5776     | 438976 |
| 53     | 2809     | 148877 | 77     | 5929     | 456533 |
| 54     | 2916     | 157464 | 78     | 6084     | 474552 |
| 55     | 3025     | 166375 | 79     | 6241     | 493039 |
| 56     | 3136     | 175616 | 80     | 6400     | 512000 |
| 57     | 3249     | 185193 | 81     | 6561     | 531441 |
| 58     | 3364     | 195112 | 82     | 6724     | 551368 |
| 59     | 3481     | 205379 | 83     | 6889     | 571787 |
| 60     | 3600     | 216000 | 84     | 7056     | 592704 |
| 61     | 3721     | 226981 | 85     | 7225     | 614125 |
| 62     | 3844     | 238328 | 86     | 7396     | 636056 |
| 63     | 3969     | 250047 | 87     | 7569     | 658503 |
| 64     | 4096     | 262144 | 88     | 7744     | 681472 |
| 65     | 4225     | 274625 | 89     | 7921     | 704969 |
| 66     | 4356     | 287496 | 90     | 8100     | 729000 |
| 67     | 4489     | 300763 | 91     | 8281     | 753571 |
| 68     | 4624     | 314432 | 92     | 8464     | 778688 |
| 69     | 4761     | 328509 | 93     | 8649     | 804357 |
| 70     | 4900     | 343000 | 94     | 8836     | 830584 |
| 71     | 5041     | 357911 | 95     | 9025     | 857375 |
| 72     | 5184     | 373248 | 96     | 9216     | 884736 |

*A Table of Squares and Cubes.*

| Roots. | Squares. | Cubes.  | Roots. | Squares. | Cubes.  |
|--------|----------|---------|--------|----------|---------|
| 97     | 9409     | 912673  | 121    | 14641    | 1771561 |
| 98     | 9604     | 941192  | 122    | 14884    | 1815848 |
| 99     | 9801     | 970299  | 123    | 15129    | 1860867 |
| 100    | 10000    | 1000000 | 124    | 15376    | 1906624 |
| 101    | 10201    | 1030301 | 125    | 15625    | 1953125 |
| 102    | 10404    | 1061208 | 126    | 15876    | 2000376 |
| 103    | 10609    | 1092727 | 127    | 16129    | 2048383 |
| 104    | 10816    | 1124864 | 128    | 16384    | 2097152 |
| 105    | 11025    | 1157625 | 129    | 16641    | 2146689 |
| 106    | 11236    | 1191016 | 130    | 16900    | 2197000 |
| 107    | 11449    | 1225043 | 131    | 17161    | 2248091 |
| 108    | 11664    | 1259712 | 132    | 17424    | 2299968 |
| 109    | 11881    | 1295029 | 133    | 17689    | 2352637 |
| 110    | 12100    | 1331000 | 134    | 17956    | 2406104 |
| 111    | 12321    | 1367631 | 135    | 18225    | 2460375 |
| 112    | 12544    | 1404928 | 136    | 18496    | 2515456 |
| 113    | 12769    | 1442897 | 137    | 18769    | 2571353 |
| 114    | 12996    | 1481544 | 138    | 19044    | 2628072 |
| 115    | 13225    | 1520875 | 139    | 19321    | 2685619 |
| 116    | 13456    | 1560896 | 140    | 19600    | 2744000 |
| 117    | 13685    | 1601613 | 141    | 19881    | 2803221 |
| 118    | 13924    | 1643032 | 142    | 20164    | 2863288 |
| 119    | 14161    | 1685159 | 143    | 20449    | 2924207 |
| 120    | 14400    | 1728000 | 144    | 20736    | 2985984 |

*A Table of Squares and Cubes.*

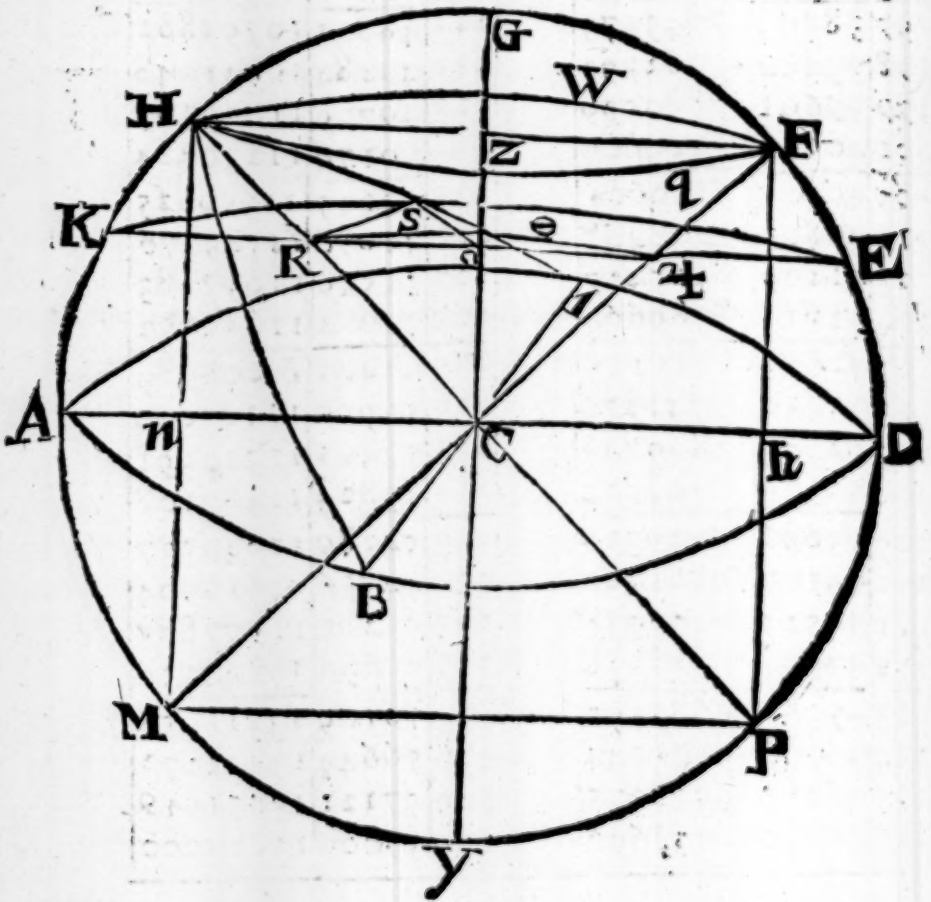
| Roots. | Squares. | Cubes.  | Roots. | Squares. | Cubes.  |
|--------|----------|---------|--------|----------|---------|
| 145    | 21025    | 3048625 | 169    | 28561    | 4826809 |
| 146    | 21316    | 3112136 | 170    | 28900    | 4913000 |
| 147    | 21609    | 3176523 | 171    | 29241    | 5000211 |
| 148    | 21904    | 3241792 | 172    | 29584    | 5088448 |
| 149    | 22201    | 3307949 | 173    | 29929    | 5177717 |
| 150    | 22500    | 3375000 | 174    | 30276    | 5268024 |
| 151    | 22801    | 3442951 | 175    | 30625    | 5369375 |
| 152    | 23104    | 3511808 | 176    | 30976    | 5451776 |
| 153    | 23409    | 3581577 | 177    | 31329    | 5545233 |
| 154    | 23716    | 3652264 | 178    | 31684    | 5639752 |
| 155    | 24025    | 3723875 | 179    | 32041    | 5735339 |
| 156    | 24336    | 3796416 | 180    | 32400    | 5832000 |
| 157    | 24649    | 3869893 | 181    | 32761    | 5929741 |
| 158    | 24964    | 3944312 | 182    | 33124    | 6028568 |
| 159    | 25281    | 4019679 | 183    | 33489    | 6128487 |
| 160    | 25600    | 4096000 | 184    | 33856    | 6229504 |
| 161    | 25921    | 4173241 | 185    | 34225    | 6331625 |
| 162    | 26244    | 4251528 | 186    | 34596    | 6434856 |
| 163    | 26569    | 4330747 | 187    | 34969    | 6539203 |
| 164    | 26896    | 4410944 | 188    | 35344    | 6644672 |
| 165    | 27225    | 4492125 | 189    | 35721    | 6751269 |
| 166    | 27556    | 4574296 | 190    | 36100    | 6859000 |
| 167    | 27889    | 4657463 | 191    | 36481    | 6967871 |
| 168    | 28224    | 4741632 | 192    | 36864    | 7077888 |

*A Table of Squares and Cubes.*

| Roots. | Squares. | Cubes.   | Roots. | Squares. | Cubes.   |
|--------|----------|----------|--------|----------|----------|
| 193    | 37249    | 7189057  | 217    | 47089    | 10218313 |
| 194    | 37636    | 7301384  | 218    | 47524    | 10360232 |
| 195    | 38025    | 7417875  | 219    | 47961    | 10503459 |
| 196    | 38416    | 7529536  | 220    | 48400    | 10648080 |
| 197    | 38809    | 7645373  | 221    | 48841    | 10793861 |
| 198    | 39204    | 7762392  | 222    | 49284    | 10941048 |
| 199    | 39601    | 7880599  | 223    | 49729    | 11089567 |
| 200    | 40000    | 8000000  | 224    | 50176    | 11239424 |
| 201    | 40401    | 8120601  | 225    | 50625    | 11390625 |
| 202    | 40804    | 8242408  | 226    | 51076    | 11543176 |
| 203    | 41209    | 8365427  | 227    | 51529    | 11697083 |
| 204    | 41616    | 8489664  | 228    | 51984    | 11852352 |
| 205    | 42025    | 8615125  | 229    | 52441    | 12008989 |
| 206    | 42436    | 8741816  | 230    | 52900    | 12167000 |
| 207    | 42849    | 8869743  | 231    | 53361    | 12326391 |
| 208    | 43264    | 8998912  | 232    | 53824    | 12487168 |
| 209    | 43681    | 9129329  | 233    | 54289    | 12649337 |
| 210    | 44100    | 9261000  | 234    | 54756    | 12812904 |
| 211    | 44521    | 9393931  | 235    | 55225    | 12977875 |
| 212    | 44944    | 9528128  | 236    | 55696    | 13144256 |
| 213    | 45369    | 9663597  | 237    | 56169    | 13312053 |
| 214    | 45796    | 9800344  | 238    | 56644    | 13481272 |
| 215    | 46225    | 9938375  | 239    | 57121    | 13651919 |
| 216    | 46656    | 10077696 | 240    | 57600    | 13824000 |

## PROPOSITION XXV.

**L** Et AYDG be a *Sphere*, **CY** its *axis*,  
**C** the *Center* of the *Sphere*, the *Lines*  
**AD** and **BI** be at right Angles, and at right  
 Angles with the *axis* **GY**.





Let this *Sphere* be cut by a *Plane* through the *axis*  $GY$ , and the *Base*  $AD$ ; the *Section* makes the *Circle*  $AYDG$ .

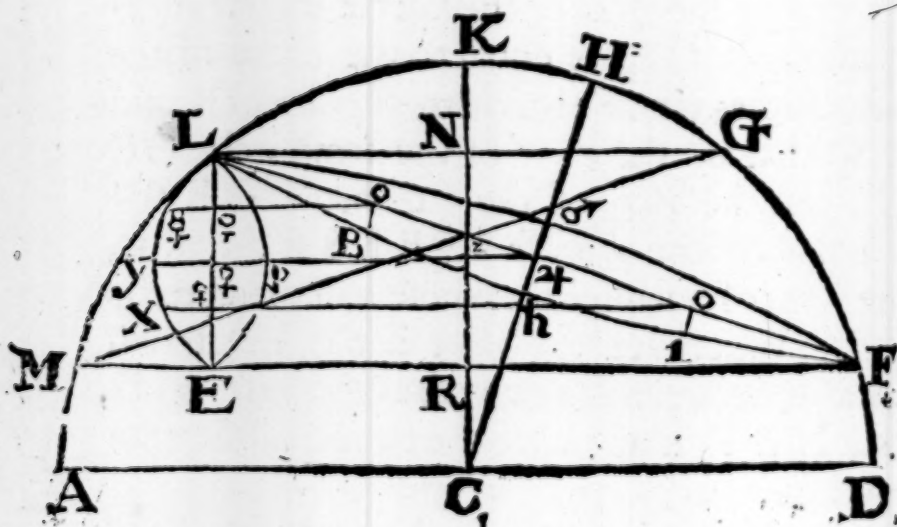
Let the *Sphere* be cut by another *Plane*, viz. as by the *Plane*  $CBHSI$ ;  $HC$  its diameter,  $BI$  its diameter of the *Base*. In the inclining *Solid* whose diameter of the *Base* is  $BI$ , and diameter of the *Section*  $CH$  and the altitude  $CZ$  or  $HN$ , that is the inclining *Solid*  $BHSIC$  is equal to the *Zone*  $AHFCD$ , less the *Cone*  $HWFAQ$ , or the inclining *Solid*  $BHSIC$  is equal to the *Excavatus* part of the *Sphere*  $ACHFCD$ . Let the *plane*  $KSE$  be parallel to the *plane*  $AID$ ;  $RS$  the common section of the *planes*  $KSE$  and  $CHI$ :  $OE$ ,  $OS$  and  $OK$  are equal; the square of  $OS$  is equal to the squares of  $RO$  and  $RS$ . 47. 1. Therefore the *Area* of the *Circle* whose semidiameter is  $OS$ , is equal to the *Areas* of the two *Circles* whose semidiameters are  $OR$  and  $RS$ , that is, the *Area* of the semicircle  $OKSE$ , less the *Area* of the semicircle  $ORQ$  is equal to the *armille*  $QORKSE$ , that is, the *Area* of a semicircle whose semidiameter is  $RS$ . Wherever the *plane*  $KSE$  be drawn parallel between the two parallel *planes*  $AID$  and  $HWF$  it will have the same qualification by the 47. 1.

Whence it is manifest the inclining *Solid*  $BHSIC$  is the difference betwixt the *Zone*  
A

**AHFD** and the *Cone* **HWFQC**. Further, this inclining *Solid* **BHSIC** is equal to a *hemispheroid* whose *axis* is **HN**, the altitude of the section **BHSIC**, and the *semidiameter* of the *Base* of the *hemispheroid* is **CA**. By the 10th. of the 6th. of *Euclid*, and by the 3d. of the 23. Proposition.

2.

Let **AKD** be a *Hemisphere*; the *Plane* **AKD** and the *Plane* **LbFδ** cutting one another at right Angles their common section the line **LF**. From the points **L** and **F** draw the Lines as **LG** and **FM** parallel to the line **AD**; also from the points **L** and **G**, to the points **F**



and



3.

Let AGE be a *Hemisphere*; the planes AGE and GIE cutting one another at right Angles; their common section the line GE. Let GZ be equal to ZE; and also RL equal to ZI; then will the *Hemisphere* AGE less the *cone* whose diameter of the *Base* is AE and Altitude CG, that is the *Cone* AGE, be equal to a *Spheroid* whose *axis* is CG and conjugate semidiameter is RL. By the 47th. 1. and 10th. and 6th. and by the third of the 23d. Prop.

4.

Let AGE be a *Hemisphere*; the Planes AGE and GδQ cutting one another at right Angles; their common section the line GB. Let G♀ be equal to ♀B being continued till it meets with the Circle GYA being continued; Let RO be equal to δ♀. Then will the *Hemisphere* AGE, less the *cone* whose diameter of the *Base* is BD and altitude CG, that is the *cone* BGD, be equal to the *frustum spheroid* whose *axis* is BP and conjugate semidiameter is OR. By the 47. 1. and 10. 6. and by the third of the 23d. Proposition.

This holds true not only in the *Sphere*, but also in both the *Spheroides*; as well in the *cone* as also in both the *Conoides*, not only when the

the cutting plane cuts the *axis*, but when it is parallel thereunto; but when it is parallel to the *axis*, there will be a *cylinder* instead of these *cones*. The *Excavatus* parts in the *sphere* and both the *spheroides*, will always be *spheres* or *spheroides*, or parts thereof their demonstrations by the 47. 1. 10. 6. and by the third of the 23d. Prop.

## 5.

The *Excavatus* parts of a *frustum cone*, will have relation to the *hyperbola*, *ellipsis* and *parabola*; Their demonstrations from the 47. 1. 10. 6. and from the first, third and fifth of the 23. Prop.

## 6.

The *Excavatus* parts of a *hyperbolick conoide* will have relation to all the three sections, *viz.* the *hyperbola*, *ellipsis* and *parabola*; their demonstrations by the 47. 1. 10. 6. and by the first, third and fifth of the 23. Prop.

## 7.

The *Excavatus* parts of a *parabolick conoide*, will have relation but to the *ellipsis* and *parabola*, their demonstrations from the 47. 1. and 10. 6. and from the third and fifth of the 23. Proposition.



8.

The solidity of every *frustum cone*, is equal to a *cylinder* whose *Base* is the lesser *Base* of the *frustum*, and its altitude the altitude of the *frustum*, more a *hyperbolick conoide* whose *Base* is equal to the difference of the *Bases* of the said *frustum*, and its altitude the altitude of the *frustum*; the Transverse Diameter of the *hyperbolick conoide* will be a line intercepted, betwixt the continuation of the other side of the *frustum cone*, and the intercepted Diameter, being continued.

9.

The solidity of every *frustum parabolick conoide*, is equal to a *cylinder* whose *Base* is the lesser *Base* of the *frustum*, and altitude the altitude of the said *frustum*, more a *parabolick conoide* whose *Base* is equal to the difference betwixt the *Bases* of the said *frustum*, and its altitude the altitude of the *frustum*.

10.

The solidity of every *frustum hyperbolick conoide*, is equal to a *cylinder* whose *Base* is the lesser *Base* of the *frustum*, and altitude the altitude of the said *frustum*; more a *hyperbolick conoide* whose *Base* is equal to the difference betwixt the *Bases* of the said *frustum*; and

and its altitude the altitude of the *frustum*.

This latter *hyperbolick conoide*, is like to that *hyperbolick conoide*, of which the *frustum* is a part.

## II.

A *Sphere* being cut by two parallel planes, both of them equidistant & parallel to a plane passing through the Center of the *Sphere*; includes a part of the *Sphere*, which for distinction sake may be called a *middle Zone*.

The solidity of every *middle Zone* of any *Sphere*, is equal to a *cylinder* of the same *Base* and altitude as the *Zone*; more a *Sphere* whose diameter is equal to the altitude of the *Zone*.

## 12.

A *Spheroid* being cut by two parallel planes, both of them equidistant and parallel to a plane passing through the Center of the *Spheroid* and cutting the *Axis* of the said *Spheroid* at right Angles, includes a part of the *spheroid*, which part for distinction sake may be called the *middle Zone* of a *spheroid*.

The solidity of the *middle Zone* of any *spheroid*, is equal to a *cylinder* of the same *Base*, and altitude as the *Zone*; more a *spheroid* whose *Axis* is equal to the altitude of the *Zone*.

The

The *ellipsis* which generates this last *spheroid* is like to that *ellipsis* which generated that *spheroid* of which this *middle Zone* is a part.

## 13.

The solidities of *hyperbolick conoides* upon the same *Base*, are as their altitudes. Here the Transverse Diameter is increased or decreased in the same proportion as the intercepted Diameter. By the first of the 23d. Proposition.

And also,

The solidities of *hyperbolick conoides* under the same altitude, are as their *Bases*. Here the conjugate Diameter is increased or decreased in the same proportion as the ordinate. By the second of the 23d. Proposition.

The solidities of like *hyperbolick Conoides*, are in a triplicate *ratio* of their corresponding terms, that is, their Transverse Diameters, or their intercepted Diameters, their conjugate Diameters, or the Ordinates.

## 14.

The solidities of *hemispheroides* upon the same *Base*, are as their Altitudes. By the 3d. of the 23d. Prop.

The solidities of *hemispheroides* under the same Altitude, are as their *Bases*, by the 4th. of the 23d. Prop. The

The solidities of like *spheroides*, are in a *triplicate ratio* of their corresponding terms.

15.

The solidities of *parabolick conoides* upon the same *Base*, are as their *Altitudes*. By the 5th. of the 23d. Prop.

The solidity of *parabolick conoides* under the same *Altitude*, are as their *Bases*. By the 6th. of the 23d. Prop.

The solidities of like *parabolick conoides*, are in the *triplicate ratio* of their corresponding terms.

*An Example thus.*

Suppose there be two *parabolick conoides*, A and B, the solidity of A, will be to the solidity of B; as the Cube of the *axis* of A, is to the Cube of the *axis* of B.

Further, as the *parabolick conoide* of A, is to the *parabolick conoide* of B; so is the Cube of *latus rectum* of A, to the Cube of *latus rectum* of B.

Yet further, if these *conoides* both equally incline, it will be;

As the *conoide* of A, is to the *conoide* of B; so will the Cube of the *Diameter* of A, be to the Cube of the *Diameter* of B. The like in the rest.

If there be two *parabolick conoids* unlike, suppose A and D. Then,

A will have that proportion to D, as is  
H com-



composed of the *Base* and altitude of A, to the *Base* and altitude of D.

In *parabolick conoides*, as A and B.

If the *conoide* of A be equal to the *conoide* of B, then their *Bases* and altitudes are reciprocal, and if their *Bases* and altitudes are reciprocal; those *conoides* are equal.

Thus.

As the *Base* of A, is to the *Base* of B; so is the altitude of B to the altitude of A.

These are said to be reciprocal, and their magnitudes are equal.

The like in *hemispheroides*, but not in *hyperbolick conoides*; for there may be infinite *hyperbolick conoides*, yet having the same *Base* & altitude; the least not so little as a *Cone*, nor the greatest so great as a *parabolick conoide* of that same *Base* and altitude. In this condition the semidiameter of the *Base*, and the altitude are supposed to be equal.

16.

Spheres of equal Diameters may be added together, their sum will be a *spheroid*, whose *axis* will be equal to the Diameter of one of the *spheres*; But the *area* of that Circle which passeth through the Center of this *spheroid*, and cutteth the *axis* at right Angles; will be equal to the *areas* of so many great Circles of those *spheres*, as there are *spheres* in number. Suppose there be six *spheres* to be added together,



ther, the *axis* of the *spheroid* will be equal to one of their *Diameters*; but the *area* of that *Circle*, that passeth through the *Center* of that *spheroid*, and cutteth the *axis* at right *Angles*, shall be equal to the *areas* of six *Circles* whose *Diameter* is equal to the *Diameter* of one of the *spheres*. The *areas* of the *Circles* are added, or subtracted, by the 47th. of the 1. of *Euclid*.

*Spheres* and *Spheroides* of the same *axis*, may be added to, or subtracted from each other, their sum, and difference will be *spheres* or *spheroides*.

## 17.

*Hyperbolick conoides* of the same *axis* may be added too, or subtracted from each other, their sum and difference will be *hyperbolick conoides*. Here you are to take the sum of the *Bases*, for the *base* of the sum, and the difference of the *bases*, for the *base* of the difference. Further, We are to take both the *parameters* for the *parameter* of the sum, and the difference of both the *parameters* for the *parameter* of the difference.

## 18.

*Parabolick conoides* of the same *axis*, may be added too, or subtracted from each other, the sum and difference will be *parabolick conoides*, using the former rules for their *bases* and *parameters*.

H 2

Here

Here note, the line P Z in the Diagram for the 9th. Propof. the line P N in the Diagram for the 10th. Propof. the line N H in the Diagram for the 11th. Propof. are called *parameters*.

## 19.

If the *axis*, Transverse and conjugate Diameters of a *hyperbolick conoide* be equal one to another; and the *axis* equal to the *axis* of a *Sphere*: this *hyperbolick conoide* and *Sphere* being added together, they make a *parabolick conoide*; its *Base* and altitude equal to the *Base* and altitude of the *hyperbolick conoide*, its *parameter* the double of the Diameter of the *Sphere*.

If there be a *hyperbolick conoide*, A; and a *Spheroid*, B; their *axis* equal: If the transverse Diameter of A, is to the *parameter* of B; as the *parameter* of A, is to the *parameter* of B. Two such *conoides* being added together, they will make a *parabolick conoide*; its *Base* the same as the *hyperbolick conoide*; its *parameter* equal to the *parameters* of the other *conoides*, being added together.

If there be a *Cone*, whose *axis* is equal to the Diameter of the *Base*; and a *Sphere* whose *axis* is equal to the *axis* of the *Cone*; this *Cone* and *Sphere* being put together, makes a *parabolick conoide*; its *Base* equal to the *Base* of  
of

of the *Cone*, its *parameter* equal to the *Diameter* of the *Sphere*:

## 20.

*Parabolick Conoides*, may always be added to, and sometimes subtracted from *hyperbolick conoides* of the same *axis*; that sum will be a *perbolick conoide*, and that difference when a difference may be, will be a *hyperbolick conoide*, the sum of their *Bases* for the *Base* of the sum: and the difference of their *Bases* for the *Base* of their difference: the sum of their *parameters* for the *parameter* of the sum, and the difference of the *parameters* for the *parameter* of the difference.

Here note, when the *parameter* of the *hyperbolick conoide* is greater than the *parameter* of the *parabolick conoide* then a difference may be: but if it be lesser, then no difference can be taken.

The Demonstrations of these are in Prop. 15, 16, 17 and 18, compared with Propos. 9, 10 and 11. of this Book.



## *The Application.*

**T**He first Proposition is to find the solidity of *pyramides* and *Caves*, or *frustum pyramides* and *Cones*, and may be applicable to the measuring of all solids or Vessels in that form, whether whole or in part, or gradually, that is, foot by foot, or inch by inch.

The second Proposition may be applied to the measuring of irregular solids, and may be useful for the exact measuring of all sorts of Stone and Timber: also for the exact measuring of all sorts of *elliptick*, *parabolick* and *hyperbolick* irregular solids, or Vessels that are made in that form: for such solids may be cut into *parallelepipedons*, *prismes* and *pyramides*, and then reduced to their own nature, by the proportion of the *parallelogram* adscribed about those Figures to the Figures themselves, Thus.

The proportion of *parallelograms* of the same *Base* and altitude with the *areas* of *parabolas* are as 4 to 3, Therefore,

As



As 4, is to 3; so is any such solid to any such *parabolick* irregular solid.

By the help of a Table of *Squares* and *Cubes* any such solids may be calculated foot by foot, or inch by inch, without any great trouble, as is shewed in the third case of the third Proposition; This second and third Propof. are the general use in such kind of solids.

In the fourth Propof. with its several cases, there is the measuring of *frustum pyramides* when their *Bases* are not parallel.

In the fifth Propof. there is the relation of the *Sphere* and *Spheroid* to the *cylinders* of their *bases* and altitudes, as well of the parts as the whole.

In the sixth Propof. there is the measuring of the *middle Zone* of a *Sphere* and *Spheroid*; the *middle Zone* of a *Spheroid*, hath been taken generally for the Figure representing a Cask; so, the measuring of one, the other is measured.

In the twelfth Propof. there is the measuring of a portion of a *Sphere*, which may be applyed to the measuring of the inverted Crown of Brewers Coppers, or several other uses.

In the thirteenth Propof. there is the measuring of *parabolick conoides*, which may be taken for a Brewers Copper, the inverted crown.



Sometimes it may be a portion of a *Sphere*, or *Spheroid*, but sometimes the portion of a *parabolick conoid*; other times the portion of a *hyperbolick conoid*, they ought to be taken as discretion seems convenient.

In the fourteenth Propos. there is the measuring of a *hyperbolick conoid*, which may be taken for a Brewers Copper.

In Propos. 15, 16, 17, and 18. there is the measuring of a *sphere*, *spheroid*, *parabolick conoid* and *hyperbolick conoid*, as well the whole as their parts.

If the *parabolick* or *hyperbolick conoids* be taken for Brewers Coppers, with the help of the Tables of Squares and Cubes, they may easily be calculated foot by foot, or inch by inch according to the third Prop.

In the twentyeth Prop. there is the measuring of Circular and *Elliptick Splindles*.

The *middle Zone* may be taken for a Cask.

In the twenty first Propos. there is the measuring of the second Section in a *sphere* and *spheroid*.

The use may be to measure the *middle Zone* of a *spheroid*, cut by a *plane* parallel to the *axis*; that is, when the *superficie* of the liquor cuts the heads of the Cask.

In the twenty fourth Propos. there is the measuring of right *cylindrick* *hoofs*, viz. *Circular*, *Elliptick*, *parabolick* and *hyperbolick*; and

and may be used for the measuring of Brewers leaning Vessels.

If a Brewers Copper be taken to be of that Figure that *parabolick* or *hyperbolick conoides* are, and they stand leaning, the measuring of them is almost the same as though they did not lean.

Here I ought to have shewed the making, and use of an Instrument for taking the leaning of such Vessels; But my business calls me off; However, they may be had of Mr. *John Marks* Instrument maker, living at the Sign of the *Ball* near *Somerset House* in the *Strand*, who was formerly Servant to that incomparable Instrument maker Mr. *Henry Sutton*.

Here note, the Table of Squares and Cubes is very ready and useful in finding the portions of a *sphere*, *spheroides*, *parabolick* and *hyperbolick conoides*.

To find two such numbers, that their Product being added to the sum of their squares, the sum shall be a square, and its Root commensurable.

Let one of the numbers be *A*, and the other *Z*. The product may be  $AZ$ ; the sum of their squares  $ZZ + AA$ ; the sum of their squares and product may be  $ZZ + AZ + AA$ , equal to a square whose side is,  $Z + A$  that

that is the square  $ZZ - 4Z + 4$ , therefore  $ZZ + ZA + AA = ZZ - 4Z + 4$ , that is,  $ZA + AA = -4Z + 4$ ; Let  $A$  be a unit, then  $5Z = 3$ , that is,  $Z = \frac{3}{5}$ , that is,  $Z$  is  $\frac{3}{5}$ , and  $A$ ,  $\frac{5}{5}$ . These two numbers make good the question, for  $3$  in  $5$ , is  $15$ ; the square of  $3$ ,  $9$ , and the square of  $5$ , is  $25$ ; their sum is  $49$ , whose Root is  $7$ . *Albert Girard* observes from this seventh Propos. of the 5th. of *Diophantus*, That if there be a Triangle made of 3 such numbers, the Angle opposite to the greatest side will be  $120$  degrees. It may be further observed, that if there be 2 right angled Triangles made of these 3 numbers, the sum of the *hypotenuse* and *base* of the one, will be equal to the sum of the *hypotenuse* and *base* of the other; and also the *area* of the one shall be equal to the *area* of the other.

Thus,

|     |     |                                  |
|-----|-----|----------------------------------|
| 749 | 749 |                                  |
| 525 | 39  |                                  |
| 74  | 58  | The sum of their Squares.        |
| 24  | 40  | The difference of their Squares. |
| 70  | 42  | Their double Rectangles.         |
| 98  | 89  | The sum of their sides.          |
| 840 | 840 | Their Areas.                     |

Here note, the double Rectangles are taken for their Altitudes.

Here

Here note,

In Progressions from a Unit, the sum, of the sum and difference of the greatest number in that progression, and any one number betwixt the greatest and Unity, is equal to the sum, of the sum and difference of that same greatest number; and any other number betwixt Unity and that greatest.

Further note,

This seventh Proposition is a *Lemma* to the eighth, to find three Triangles of equal *areas*; therefore the *areas* are equal, and the *hypotenuse* and *base* of the one, is equal to the *hypotenuse* and *Base* of the other.

A general Theorem for the finding of two such numbers; Take the square of any number, from which take a Unit; take the double of that number, to which adde a Unit; that sum and difference will be the two numbers required. Thus,

The number taken is 3, its square 9, less a unit is 8; the double of 3, is 6 more, a unit is 7; these two numbers makes good the question.

For  $56 + 64 + 49 = 169$ , whose root is 13.

Two right angled Triangles being made of these three numbers, *viz.* 7, 8, 13. according to the former method, will have that same qualification.

This



This Proposition was publicly proposed in  
 PARIS in the year 1633, which *Renatos de*  
*Cartes* resolved, and *Francis Schooten* publish-  
 ed in Sect. 12. *Miscel.* incumbred with a square  
 affected equation, with *surds*.

## F I N I S.

## Errata.

Page 17, line 3, for *Ecliptick* read *Elliptick*. p. 25, for *Excav-*  
*us* r. *Excavatus*. p. 29, l. 12, for  $\frac{1}{2}$ , r.  $\frac{1}{6}$ . In the same line, for BC  
 BFE; r. BCBFD. p. 30, l. 22, for ZB, r. ZC. p. 32. l. 19  
 for NR, r. NP. p. 39, l. 13, put ; after Line. p. 46, l. 21  
 for 634, r. 624. p. 53, l. 3, for APR, r. APK. p. 48, l. 25  
 for ZB, r. XB. p. 60, l. ult. for R  $\odot$   $\varphi$   $\psi$ , r. R  $\odot$   $\delta$   $\psi$ . p.  
 61, l. 6, for V, r. VI. p. 64, l. 4, for GG, r. GC. p. 66  
 l. 18, for CDQ, r. CGQ. p. 70, p. 71 l. 5, put , after D.  
 p. 74, l. 4, for GFPM; r. CFPM. p. 80, l. 21, put : before  
 all. p. 82, l. 25, after KABO put ,

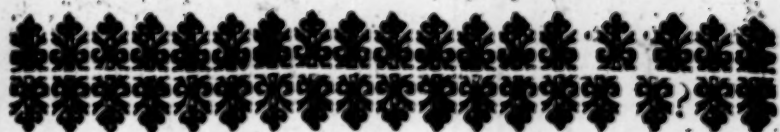
## Imprimatur.

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*Cant.* à Sac. Dom.

Ex Ad. Lamb.

Aug. 25. 1668.





# Gaging Promoted

AN

# APPENDIX

TO

## Stereometrical Propositions.

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### I. *Note.*

**A**S an Abstract from the undoubted Axioms of Geometry, it is generally observed, that in a Rank of numbers, having equal difference, the second differences of the squares of those numbers are equal; the third differences of the cubes of those numbers are equal: and so in order in the higher powers. Thus,

A 2

In

(2)

In Squares

| 1 |    |   |   |
|---|----|---|---|
| 1 | 1  | — | — |
| 2 | 4  | 3 | 2 |
| 3 | 9  | 5 | 2 |
| 4 | 16 | 7 | 2 |
|   | 25 | 9 | — |
| 1 | 2  | 3 | 4 |

| 2 |    |    |   |
|---|----|----|---|
| 1 | 1  | 8  | — |
| 3 | 9  | 16 | 8 |
| 5 | 25 | 24 | 8 |
| 7 | 49 | 32 | 8 |
| 9 | 81 | —  | — |
| 1 | 2  | 3  | 4 |

Observe in the first of these Examples, in the first collum are the numbers of a progression, having equal difference, to wit, a unite. In the second collum, the squares of those numbers. In the third collum, the first differences. In the fourth collum, the second differences, to wit, 2, 2, 2. In the second Example, in the first collum are a Rank of numbers, having equal difference, to wit, 2. In the second collum, their squares. In the third collum, the first difference. In the fourth collum, the second difference.

II. Note.

Hence it follows, that by the help of such differences the table of squares may be calculated: thus, in the first Example, the sum of 1 and 3, is 4; the square of 2. The sum of 2, 3 and 4, is 9; the square of 3. The sum of 2, 5 and 9, is 16; the square of 4. The sum of 2, 7 and 16, is 25; the square of 5. The sum of 2, 11 and 36, is 49; the square of 7. &c.

III.

(3)

## III. Note.

Like plain numbers are in the same proportion one to another, that a square number is in, to a square number: *Euclide* the 26 Proposition of the Eighth Book. Therefore the second difference in such a Rank of plane numbers are equal. Further, what planes and solids are either equal or proportionable to such Ranks may be gradually calculated; as in the last.

## IV. Note.

| I | I   |    |    |   |
|---|-----|----|----|---|
| 2 | 8   | 7  | 12 |   |
| 3 | 27  | 19 | 18 | 6 |
| 4 | 64  | 37 | 24 | 6 |
| 5 | 125 | 71 |    |   |
| 1 | 2   | 3  | 4  | 5 |

| I | I   |     |     |    |
|---|-----|-----|-----|----|
| 3 | 27  | 26  | 72  |    |
| 5 | 125 | 98  | 120 | 48 |
| 7 | 343 | 218 | 186 | 48 |
| 9 | 729 | 386 |     |    |
| 1 | 2   | 3   | 4   | 5  |

In the first Example, in the first column are the numbers in a Rank having equal difference, to wit, a unite. In the second column, the cubes of those numbers. In the third column, the first differences of those cubes. In the fourth column, the second differences. In the fifth column, the third differences, to wit, 6, 6. The like in the second Example.

## V. Note.

Hence it follows, that the table of cubes may be  
A 3 made,

(4)

made thus : In the first Example,  $\bar{1}$  and  $\bar{7}$ , is 8; the cube of 2. The sum of 8 and 19, is 27; the cube of 3. The sum of 18, 19 and 27, is 64; the cube of 4. The sum of 6, 18, 37 and 64, is 125; the cube of 5. The sum of 6, 24, 61 and 125, is 216; the cube of 6. The sum of 6, 30, 91 and 216, is 343; the cube of 7. The like in the second Example.

## VI. Note.

Like solid numbers are in the same proportion one to another, that a cube number is in, to a cube number. *Euclide* the XXVII Prop. of the Eighth Book. Therefore the third differences in such a Rank of solid numbers are equal : further, such planes and solids as are either equal or proportionable to such Ranks, may be gradually calculated, as in the last.

## VII. Note.

If a Rank of Squares, whose Roots have equal differences, be multiplied by any number, the second differences of such a Rank of products are equal. Let the number multiplying be 10.

|   |    |     |    |    |
|---|----|-----|----|----|
| 0 | 0  | 00  |    |    |
| 1 | 1  | 10  | 10 | 20 |
| 2 | 4  | 40  | 30 | 20 |
| 3 | 9  | 90  | 50 | 20 |
| 4 | 16 | 160 | 70 | 20 |
| 5 | 25 | 250 | 90 |    |
| 1 | 2  | 3   | 4  | 5  |

In the first column are the numbers bearing equal difference. In the second column are the squares of those numbers. In the third column the products. In the fourth column the first differences. In the fifth column the second differences and they equal.

## VIII. Note.

If unto such a Rank of Products, as in the last, there be added a Rank of Cubes, whose Roots are equal to the Roots of the Squares, the third differences of such a Rank will be equal.

| 0 | 00  | 0   | 00  |     |    |   |
|---|-----|-----|-----|-----|----|---|
| 1 | 10  | 1   | 11  | 11  | 26 |   |
| 2 | 40  | 8   | 48  | 37  | 32 | 6 |
| 3 | 90  | 27  | 117 | 69  | 38 | 6 |
| 4 | 160 | 64  | 224 | 107 | 44 | 6 |
| 5 | 250 | 125 | 375 | 151 |    |   |
| 1 | 2   | 3   | 4   | 5   | 6  | 7 |

In the first column are the numbers having equal difference. In the second column the Products of their squares by a given number. In the third column the cube of the numbers in the first column. In the fourth column the sum of the products and cubes. In the fifth column their first difference. In the sixth column the second differences. In the seventh column the third differences which are equal.

## IX. Note.

Let a constant number be added to a Rank of Products, so that one of the numbers multiplying  
A 4. be



be a constant number, and the other of the numbers be the squares of numbers having equal difference, and this Rank of sums be added to a Rank of cubes, whose roots are the same with the roots of the squares: such a compounded Rank will have their third difference equal. Thus,

|   |   |     |     |     |     |    |   |
|---|---|-----|-----|-----|-----|----|---|
| 1 | 8 | 10  | 1   | 19  | —   | —  | — |
| 2 | 8 | 40  | 8   | 56  | 37  | —  | — |
| 3 | 8 | 90  | 27  | 125 | 69  | 32 | 6 |
| 4 | 8 | 160 | 64  | 232 | 107 | 38 | 6 |
| 5 | 8 | 250 | 125 | 383 | 151 | 44 | — |
| 1 | 2 | 3   | 4   | 5   | 6   | 7  | 8 |

In the first column are the numbers having equal difference. In the second column is the constant number to be added. In the third column are the Rank of products, that is, the squares of the numbers in the first column multiplied by a given number. In the fourth column are the cubes of the numbers in the first column. In the fifth column are the sum of the numbers in the second, third and fourth columns. In the sixth column are the first differences of their sums. In the seventh column are the second differences. In the eighth column the third differences, and they equal.

### X. Note.

In a Rank of numbers, having equal difference, and equal in number; if the third part of the cubes of each of these numbers, be subtracted from the products of the squares of each of these numbers, in half the greatest number of that Rank, the remainders

## (7)

mainders will be a Rank of numbers, equal to all the squares in the several portions of one fourth of a sphere, whose diameter is equal to the greatest number in that Rank, and the third differences of this Rank of portions are equal; but the first and second differences will increase and decrease, differently one from another. Thus,

|    |      |      |      |     |     |    |
|----|------|------|------|-----|-----|----|
| 0  | 00   | 00   | 00   | —   | —   | —  |
| 3  | 09   | 108  | 99   | 99  | 162 | —  |
| 6  | 72   | 432  | 360  | 261 | 108 | 54 |
| 9  | 243  | 972  | 729  | 369 | 54  | 54 |
| 12 | 756  | 1728 | 1152 | 423 | 00  | 54 |
| 15 | 1125 | 2700 | 1575 | 423 | 54  | 54 |
| 18 | 1944 | 3888 | 1944 | 369 | 108 | 54 |
| 21 | 3087 | 5292 | 2205 | 261 | 162 | —  |
| 24 | 4608 | 6912 | 2304 | 96  | —   | —  |
| I  | 2    | 3    | 4    | 5   | 6   | 7  |

In the first column are the numbers having equal difference. In the second column are the pyramides adscribed within the cubes of the numbers in the first column. In the third column are the products of the squares of the numbers in the first column, by half the greatest number in the first column. In the fourth column are the differences of the numbers in the second and third columns, that is, all the squares in several portions of one fourth of a sphere, whose diameter is 24. In the fifth column are the first differences. In the sixth column are their second differences. In the seventh column are the third differences, and they equal.

## XI. Note.

*The Application or Use of the  
Preceeding Notes.*

The application or Use may be, to calculate Pyramides and cones, either the whole or their parts; as also to calculate the parabolick and hyperbolick conoides, either the whole, or their frustums; yet also, to calculate the sphere or spheroides, either the whole or their portions or Zones, and that gradually, that is, to find the solidity upon every inch or foot.

*To find the solidity of a parabolick Conoide upon  
every two inches,*

To do which, consider the Diagram of the 18 Prop. of my *Stereometrical Prop.* Let  $PA$  be 16;  $AR$  12; therefore  $AV$  or  $PH$  will be 9; for it ought to be as  $PA$ , is to  $AR$ ; so is  $AR$ , to  $AV$ . Let the axis  $AP$  be divided into eight equal parts; viz. 2, 4, 6, 8, 10, 12, 14, 16. Let there be planes drawn parallel to the base, through every one of these divisions, though in the Diagram there is not so many. From  $P$  to the first  $Q$  suppose to be 2, its square 4; the half thereof 2, which multiplied by 9 equal to  $PH$ , the Product will be 18; that is, the Prism  $QZGIHP$ ; equal to all the squares in the portion of the conoid  $QOP$ . Let from  $P$ , to the second  $Q$  be 4, its square 16, the half is 8; which multi-

(9)

multiplied by 9, the Product is 72; equal to the Prism QZFIHP, equal to all the squares in the portion of the conoid QOP. Let from P to the third Q be 6, its square 36, the half of it is 18, which multiplied by 9, the product is 162; the Prism QZEIHP; equal to all the squares in the third portion of the conoid QOP.

Having obtained two portions, the rest may be obtained thus: having obtained the second difference, which is 36, we may proceed to find the rest by the *Seventh Note*; thus, add 36 to 54, and it makes 90: which added to 72, the sum is 162, equal to all the squares in that portion, and so in order; 36, 90 and 162; the sum is 288. 36, 126 and 288; the sum is 450. 36, 162 and 450, the sum is 648, &c.

|    |      |     |    |
|----|------|-----|----|
| 0  | 00   |     |    |
| 2  | 48   | 8   | 36 |
| 4  | 72   | 54  | 36 |
| 6  | 162  | 90  | 36 |
| 8  | 288  | 126 | 36 |
| 10 | 450  | 162 | 36 |
| 12 | 648  | 198 | 36 |
| 14 | 882  | 234 | 36 |
| 16 | 1152 | 270 |    |
| 1  | 2    | 3   | 4  |

In the first column are the parts of the altitude of the conoid. In the second column are all the squares in several portions of one fourth of a conoid. In the third column the first differences. In the fourth column the second differences. These portions in the second column may be reduced to circular portions, thus, as 14 is to 11; so are all the squares in these portions

to the portions themselves,

## I. Scholium :

The use of this gradual calculation may be thus: Suppose a Brewers Copper be in form of a parabolick



bolick conoid; the quantity of liquor therein contained may be found, thus, having calculated a table upon every inch, or two inches, or as is thought convenient; then having a straight Ruler divided equally into inches, putting the Ruler into the liquor to the bottom of the Copper, see how many inches of the Ruler is wet; with the number of wet inches enter the first colum of your table, and in the next colum are the number of cubick inches which that portion contains; the number of cubick inches thus found, being divided by the number of cubick inches in a Gallon, the quotient shews the number of Gallons in that portion of the Copper.

## II. Scholium:

*To compose several works into one,*

As 14, is to 11; so are all the squares in one fourth of the conoid, to one fourth of the conoid it self. because this one fourth ought to be divided by the number of cubick inches in a Gallon, suppose it 288, to shew the number of Gallons in each portion, we may multiply 14 by 288, that is 4032. Then as 4032 is to 11; so are all the squares in one fourth of the conoid, to the Gallons in that one fourth. Further, because this one fourth ought to be multiplied by 4, to reduce it to a whole conoid; therefore, divide the constant divisor, that is, 4032, by 4, and it will be 1008. Then, as 1008, is to 11; so are those several portions in the second colum of the last table, to the num-



( II )

number of Gallons in those several portions of a parabolick conoid. By such compositions may the Practitioner compose constant divisors or dividends, which will much breviate the work; this is onely for an Example.

Every parabolick conoid hath its second differences equal. To find the second differences, work thus, Square one of the equal segments of the axis, and multiply that Square by the Parameter, that product will be the second difference. In this Example, the equal segment of the axis is 2, the square of it 4; which multiplied by the Parameter 9, the product is 36, the second difference. Half of the second difference, is always the first of the first difference. Half 36 the second difference, is 18, the first of the first difference, &c. Here note, this 36 is the second difference of one fourth of all the squares in a parabolick conoid; if 36 be multiplied by 4, it makes the second difference 144; whose half is 72, the first of the first differences. Or, the first differences are found by taking half the difference of the squares of any two segments, which multiplied by the Parameter, thereby the first differences are obtained. Thus, to find the first difference answerable to the Segments 6 and 8; the Square of 8, is 64; the Square of 6, is 36; the difference of those Squares is 28, whose half is 14; which multiplied by the Parameter 9, the product is 126; the first difference answerable betwixt 6 and 8.

## XII. Note.

*To find the solidity of an hyperbolick conoid gradually, to wit, upon every three inches.*

For the performance of which, take notice of the XVI Prop. of *Stereom. Prop.* in that Diagram, Let  $AM$  equal to  $AB$  be 9. Let  $ML$  equal to  $AF$ , be 6. Let  $AE$  be 15 : therefore  $FE$  will be 9. Let the rest of the construction be as in that Proposition. Let from  $M$  to the first  $K$  be 3, whose square is 9, whose half is 4½, the area  $KHM$ , which being multiplied by  $ML$ , 6; the product will be 27, the prism  $KHNOLM$ . Because  $FE$ ,  $FC$  and  $FL$  are equal, that is, each of them 9 : Therefore, the first pyramid  $ONXILM$  will be 9. Then this prism and pyramid being added, will make 36, the whole prism  $KHXILM$ , equal to all the squares in the portion  $KZM$ . Let from  $M$  to the second  $K$  be 6, whose square is 36, its half 18, the area  $KHM$  : which being multiplied by  $ML$ , 6; the product will be 108, equal to the prism  $KHNOLM$ . The cube of 6, is 216; a third part is 72, the pyramid  $ONXIL$ , this prism and pyramid being added together, the sum will be 180; the prism  $KHXILM$  : equal to all the squares in the portion  $KZM$ . Let from  $M$  to  $A$ , be 9; its square 81, the half 40½, equal to the area  $ABM$ , which being multiplied by  $ML$ , 6; the product will be 243 : the cube of 9, is 729; a third part thereof is 243; equal to the pyramid  $FCDEL$  : this prism and pyramid being added together, is 486; the whole

whole prism **A B D E L M**, equal to all the squares of the portion **A Z M**.

These three portions being obtained, they may be continued by the **VIII Note**, thus :

|    |      |      |     |    |
|----|------|------|-----|----|
| 0  | 00   | 36   | 108 | —  |
| 3  | 46   | 144  | 162 | 54 |
| 6  | 180  | 306  | 216 | 54 |
| 9  | 486  | 522  | 270 | 54 |
| 12 | 1008 | 792  | 324 | 54 |
| 15 | 1800 | 1116 | 378 | 54 |
| 18 | 2216 | 1494 | 432 | 54 |
| 21 | 4410 | 1926 |     | —  |
| 24 | 6336 |      |     | —  |
| 1  | 2    | 3    | 4   | 5  |

For if the third differences which are equal, and in this Example is 54, be added to the first of the second differences, being 108, it makes 162, and by such additions, the second differences in the fourth column are made. Further, by adding these second differences to the first of the first differences which is 36, it makes

144, &c. So the numbers in the third column are made. Yet further, by adding these first differences to the first number in the second column, the Rank of portions of such a conoid is made.

Then,

By making use of the directions in the first and second *Scholiums*, the number of Gallons are obtained. The parabolick and hyperbolick conoides may well be made use of for Brewers Coppers; the parabolick, when the crown is somewhat blunt; but the hyperbolick conoid when the crown is more sharp.

## XIII. Note.

*To calculate a sphere gradually, to wit, upon every three Inches.*

Consider the XV Prop. of *Stereom. Prop.* Let ED equal to EF, be 24. The rest of the construction as in that Prop. Let from E, to the first R be 3, whose square is 9; whose half is  $4\frac{1}{2}$ , the area R X E, which being multiplied by 24, the product will be 108; the prism K H X R E F. The cube of 3, is 27; a third part thereof is 9, the pyramid K H O I F; this pyramid taken from the former prism, leaves the prism R X O I F E, 99 : equal to all the squares in the portion R Q E. From E, to the second R, 6; its square 36, the half 18, which multiplied by 24, makes 432; the prism R X H K F E. The cube of 6, is 216, a third part of it is 72; the pyramid K H O I F : this pyramid being taken from that prism, there rest 360; the prism R X O I F E, equal to all the squares in the portion R Q E. Let from E to the third R be 9; its square 81, the half thereof  $40\frac{1}{2}$ , the area R X E, this area being multiplied by 24, the product will be 972, the prism K H Y R E F : the cube of 9, is 729, a third part of it is 243, the pyramid K H O I F : this pyramid being subtracted from that prism, the remainder is 729; the prism R X O I F E, equal to all the squares in the third portion R Q E. Having obtained these three portions, the rest may be found by their third difference, according to the X. Note.

The



|    |      |      |      |     |     |    |
|----|------|------|------|-----|-----|----|
| 0  | 00   | 0    | 0    | —   | —   | —  |
| 3  | 09   | 108  | 99   | 99  | 162 | —  |
| 6  | 72   | 432  | 360  | 261 | 108 | 54 |
| 9  | 243  | 972  | 729  | 369 | 54  | 54 |
| 12 | 576  | 1728 | 1152 | 423 | 00  | 54 |
| 15 | 1125 | 2700 | 1575 | 423 | 54  | 54 |
| 18 | 1944 | 3888 | 1944 | 369 | 108 | 54 |
| 21 | 3087 | 5292 | 2205 | 261 | 162 | 54 |
| 24 | 4608 | 6912 | 2304 | 99  | —   | —  |
| 1  | 2    | 3    | 4    | 5   | 6   | 7  |

The numbers in the seventh column are the third differences, and they equal; the numbers in the sixth column are the second differences, and are composed by subtracting the numbers in the seventh from the first and last numbers in the sixth column; the numbers in the fifth column are the first differences, and are composed by adding those numbers in the sixth column to the first and last of those in the fifth column; the numbers in the fourth column are all the squares in several portions of one fourth of a sphere whose diameter is 24, those portions are made by adding the numbers in the fifth column to the numbers in the fourth, thus, 261, and 99, is 360. 369, and 360, is 729. 423, and 729, is 1152, &c.

Then making use of the first and second scholium the number of gallons are obtained. Or if it be made, as 14, is to 11, so is 54, to a fourth number,

B



number, with that fourth number proceede to make tables of the second and first differences, and then the table of portions it selfe. Every sphere hath its third differences equall. To find the third difference, doe thus. Cube one of the equal segments of the axis and multiply that cube by 2, and that product will be the third difference, thus, the cube of three is 27, which multiplied by 2, the product is 54; the third difference of all the squares in one fourth of a sphere. Here note, that it is to be understood, that the axis of the sphere is equally divided into an equal number of segments; so then, if the number of segments in the semiaxis, less by one; be multiplied by the third difference, it gives the first of the second differences. Thus, the number of segments in the semiaxis is 4, then 4 less 1, is 3; which being multiplied by 54, the product is 162: the first of the second differences.

To find the third difference in one fourth of all the squares in a spheroid, do thus: The axis being divided as above in the sphere; cube the difference betwixt two Segments, which being multiplied by 2, makes a product; then, as the square of the semiaxis, is to the square of the other semidiameter; so is that former product to a fourth number, which will be the third difference. For the second differences, use the Rules given for the sphere.

#### XIV. *Note.*

To calculate a pyramid or cone gradually. To find the third difference in a pyramid work thus,  
the

the Altitude of the pyramid being equally divided, cube the difference of the two segments, which being doubled, makes a number ; then, as the square of the Altitude of the pyramid, is to the area of the base of that pyramid ; so is that former number, to the third difference of that pyramid.

To find the second differences in a pyramid : As the difference of two of the segments of the Altitude, is to the following segment ; so is the third difference, to the second difference answerable to that segment.

To find the first differences in a pyramid. Cube two of the segments, and take a third part of their difference. Then, as the square of the Altitude of the pyramid ; is to the area of the base of that pyramid ; so is that former difference ; to the first difference answerable to those two segments.

Let there be a pyramid whose Altitude is 10, and one side of the base is 40, and the other side 5 ; therefore the area of the base is 200. Let the Altitude be divided into five equall parts, and to calculate accordingly. To find the third difference, the cube of 2, is 8 ; whose double is 16. Then, as 100 the square of the Altitude, is to 200 the area of the base ; so is 16, to the third difference 32. To find any of the second differences at demand, to find the second difference answerable to 8. As 2, the difference betwixt the segments 6, and 8, is to 8 ; so is the first difference 32, to 128 the second difference answerable to 8. The second differences are in proportion one to another, as their answering segments ; as 2, is to

$32$ ; so is  $8$ , to  $128$ . To find any of the first differences, cube the two Segments, to wit,  $2$  and  $4$ , and the cubes will be  $8$  and  $64$ ; then take  $8$  from  $64$  and the Remainder is  $56$ , a third part is  $18\frac{2}{3}$ . then, as the square of the Altitude  $100$ , is to the area of the base  $200$ ; so is  $18\frac{2}{3}$ , to  $37\frac{1}{3}$ , the first difference answering to  $2$  and  $4$ . Then by a continuall adding of the third difference to the second differences they are made, and by adding the first of the second differences to the first of the first differences and so in order the first differences are made. Lastly by adding the first differences the Segments of the pyramid, are made according to the III. Note; or thus.

|    |                  |                  |     |    |
|----|------------------|------------------|-----|----|
| 0  | 0                | 0                | —   | —  |
| 2  | $5\frac{1}{3}$   | $37\frac{1}{3}$  | 32  | —  |
| 4  | $42\frac{2}{3}$  | $101\frac{1}{3}$ | 64  | 32 |
| 6  | 144              | $197\frac{1}{3}$ | 96  | 32 |
| 8  | $241\frac{1}{3}$ | $325\frac{1}{3}$ | 128 | 32 |
| 10 | $666\frac{2}{3}$ | —                | —   | —  |
| 1  | 2                | 3                | 4   | 5  |

The numbers in the fifth column are the third differences, the first number in the fourth column being found by the Rule before given, all the numbers in that fourth column may be made by adding the third difference, thus, to  $32$  adde  $32$ , the summe is  $64$ . adde  $32$ , to  $64$ ; the summe is  $96$ . adde  $32$ , to  $96$ ; the summe is  $128$ . The first number in the third column being found by the Rule above, then  $5\frac{1}{3}$  added to  $32$ ; the summe is  $37\frac{1}{3}$ .  
adde

# ( 19 )

adde 64, to  $37\frac{1}{3}$ ; the summe is  $101\frac{1}{3}$ . adde 96, to  $101\frac{1}{3}$ , the sum is  $197\frac{1}{3}$ . adde 128, to  $197\frac{1}{3}$ ; the sum is  $325\frac{1}{3}$ . further, adde the first of the third colum, to the first of the second colum; thus, adde  $5\frac{1}{3}$ , to 0; the sum will be  $5\frac{1}{3}$ , adde  $57\frac{2}{3}$ , to  $5\frac{1}{3}$ , the sum is  $42\frac{2}{3}$ , adde  $101\frac{1}{3}$ , to  $42\frac{2}{3}$ , the sum is 144. adde  $197\frac{1}{3}$ , to 144; the sum is  $341\frac{1}{3}$ , adde  $325\frac{1}{3}$ , to  $341\frac{1}{3}$ ; the sum is 666 $\frac{2}{3}$ .

It it be to calculate a cone whose diameters of the base are 40 and 5. Let it it be made, as 14, is to 11; so is 32, to the third difference of the same cone. Then proceede with the third difference to make the second and first; and lastly, the table it self.

## XV. Note.

The calculation of frustum pyramides whose bases are unlike, To the performance of which consider the third case of the second proposition of *Stereom. Prop.*

Every such solide hath its third differences equall, but the second and first differences will be complicated according to the IX. Note.

To find the third difference proper to the pyramid BCDHF, Let the construction and numbers be the same as in that diagram, and let it be to calculate it upon every two inches, thus. The cube of 2, is 8; the double thereof is 16, Then, as the square of the Altitude 40, that is 1600, is to the area of the base BCDH, 336; so is 16, to  $3\frac{36}{16}$ . by the Rule delivered in the

B 3

14 Note,



14 Note, the first of the second differences is  $3\frac{36}{100}$ . and the first of the first differences is  $\frac{36}{100}$ .

The solide HDEGVF hath its second differences equall by the VII. Note.

To find its first and second differences. The square of 2, is 4. which multiplyed by FV, 26; the product will be 104. then, as 40 the Altitude, is to HD, 28; so is 104, to  $72\frac{80}{100}$ . the second difference. Therefore the first of the first differences will be  $36\frac{40}{100}$ .

To find the second differences of the solide ABMOIF the square of 2 is 4, which multiplyed by IF, 30; the product is 120, then, as 40, the Altitude; is to OA, 12: so is 120, to 36, the second difference. Therefore the first of the first differences are 18. For the complication of these differences.

| 1                  | 2                   | 3                 |                      |
|--------------------|---------------------|-------------------|----------------------|
| $\frac{36}{100}$   | $3\frac{36}{100}$   | $3\frac{36}{100}$ | in the pyramid BCDHF |
| $36\frac{40}{100}$ | $72\frac{80}{100}$  |                   | in the prism HGEDFV  |
| 18                 | 36                  |                   | in the prism ABHOIF  |
| $54\frac{96}{100}$ | $112\frac{16}{100}$ | $3\frac{36}{100}$ | their summe.         |

Rejecting the denominators they may be written Thus,

$$| \quad 5496 \quad | \quad 11216 \quad | \quad 336 \quad |$$

Because the denominators are Rejected, therefore the two last figures toward the Right hand are decimals.



|    |         |        |       |     |
|----|---------|--------|-------|-----|
| 0  |         | 161496 |       |     |
| 2  | 161496  | 172712 | 11216 |     |
| 4  | 334208  | 184264 | 11552 | 336 |
| 6  | 518472  | 196152 | 11888 | 336 |
| 8  | 714624  | 208376 | 12224 | 336 |
| 10 | 923000  | 220936 | 12560 | 336 |
| 12 | 1143936 | 233832 | 12896 | 336 |
| 14 | 1377768 | 247064 | 13232 | 336 |
| 16 | 1624832 | 260632 | 13568 | 336 |
| 18 | 1885464 | 274536 | 13904 | 336 |
| 20 | 2160000 | 288776 | 14240 | 336 |
| 22 | 2448776 | 303352 | 14576 | 336 |
| 24 | 2752128 | 318264 | 14912 | 336 |
| 26 | 3070392 | 333512 | 15248 | 336 |
| 28 | 3403904 | 349096 | 15584 | 336 |
| 30 | 3753000 | 365016 | 15920 | 336 |
| 32 | 4118016 | 381272 | 16256 | 336 |
| 34 | 4499288 | 397864 | 16592 | 336 |
| 36 | 4897152 | 414792 | 16928 | 336 |
| 38 | 5311944 | 432056 | 17264 | 336 |
| 40 | 5744000 |        |       |     |
| 1  | 2       | 3      | 4     | 5   |

The construction of the table may be thus; the numbers in the first column are the third differences. The first number in the fourth column is the complicated second difference, and the other number in that fourth column are made thus, to the first 11216, adde 336; the sum is 11552. Then to that 11552, adde 336; the sum is 11888, &c.

The first number in the third column is complicated from the first complicated difference and a parallelepipedon whose base is the plane RIFV, and the Altitude the first Segment of the Altitude of the frustum, thus, the plane RIFV, is 780; which being doubled is 1560; then, 156000 more 5496 is 161496; the first of the first differences, then 161496 more 11216, is 172712. Further, 172712 more 11552, is 184264. Yet farther 184264 more 11888, is 196152, &c.

The numbers in the second column are made thus, the first number in the second column, is the same as the first number in the third column, then, 161496 more 172712, is 334208, and 334208 more 184264, is 518472, &c.

Then making use of the first and second scholium, the quantity of Liquor that such vessels contain may easily be obtained.

## XVI. Note.

To calculate Elliptick solides whose bases are unlike. The calculation of such solides are the same as in the 15, note for if the first, second and third complicated differences be found, then making use of this propotion as 14, is to 11; so is 336; to the third difference. And

And

As 14, is to 11; so is 11216, to the first of the second differences.

Further

As 14, is to 11; so is 161496, to the first of the first differences, then proceede to make the table it self, as in the 15 note. Or make use of the second scholium of the 11 note and you will have the quantity in Gallons.

Or

Such Elliptick solides may be calculated by the 12 note: for every such Elliptick solide is equall to a frustum hyperbolick conoide whose circular bases of the conoide, are equall to the Elliptick bases of the Elliptick solide; and the Altitude of one frustum is equall to the Altitude of the other.

## XVII. Note.

Every hyperbolick conoid hath its third differences equal.

To find the third, second and first differences in an hyperbolick conoid, and consequently to calculate that conoid gradually. In the forementioned diagram of the 17. prop. Stereom. Prop. Let GM, the Transverse diameter be 12. ML, the parameter 6. MA, the axis of the conoid 24.

To

*To calculate the solidity of this conoid  
upon every three Inches.*

*To find the third difference of this conoid.*

Take the difference of two of the Segments, to wit, 3; whose cube is 27: whose double is 54. Then, as GM, 12; is to ML, 6: so is 54, to 27. The third difference of all the squares in one fourth of that conoid proper to that pyramid FCDEL.

By the Rule in the last note the first of the second differences is 27.

For the first of the first differences, worke thus; take the first Segment which is 3; whose cube is 27, a third part is 9, then, as GM, 12; is to ML, 6: so is 9, to  $4\frac{1}{2}$ . the first of the first differences proper to the pyramid FCDEL.

The second and first differences of all the squares in one fourth of this conoid, is complicated from the second and first differences of the pyramid FCDEL, and the second and first differences of the prism ABCFLM. Every such prism hath it second difference equall.

*To find the second and first difference of the  
prism ABCFLM.*

Square the difference of two of the Segments of the axis, to wit, 3; that is 9, which being multiplied by the parameter ML, 6; the product is 54, the second difference. The first of the first differences of every such prism is half of the  
second

second difference ; therefore the first of the first differences is 27.

*To complicate these differences.*

| 1               | 2  | 3  | differences                                                  |
|-----------------|----|----|--------------------------------------------------------------|
| $4\frac{1}{2}$  | 27 | 27 | in the prism FCDEL.                                          |
| 27              | 54 |    | in the prism ABCFLM.                                         |
| $31\frac{1}{2}$ | 81 | 27 | in all the squares of one fourth of that hyperbolick conoid. |

|    |                   |                   |     |    |
|----|-------------------|-------------------|-----|----|
| 0  | 0                 |                   |     |    |
| 1  |                   | $31\frac{1}{2}$   |     |    |
| 3  | $31\frac{1}{2}$   |                   | 81  | —  |
| 6  | 144               | $112\frac{1}{2}$  | 108 | 27 |
| 9  | $364\frac{1}{2}$  | $220\frac{1}{2}$  |     | 27 |
| 12 | 720               | $355\frac{1}{2}$  | 135 | 27 |
| 15 | $1237\frac{1}{2}$ | $517\frac{1}{2}$  | 162 | 27 |
| 18 | 1944              | $706\frac{1}{2}$  | 189 | 27 |
| 21 | $2866\frac{1}{2}$ | $922\frac{1}{2}$  | 216 | 27 |
| 24 | 4032              | $1165\frac{1}{2}$ | 243 | —  |
| 1  | 2                 | 3                 | 4   | 5  |

In the first column are the third differences. In the fourth column the second differences. In the third column the first differences. In the second column the portions of all the squares of one fourth of an hyperbolick conoid, upon every three inches,



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inches, whose Transverse diameter is  $12\frac{1}{2}$ , and parameter is 6, and axis is 24.

The construction of this table is the same as the former; thus, 81 more 27; is 108. more 27; is 135. more 27; is 162. &c.

$31\frac{1}{2}$ . more 81; is  $112\frac{1}{2}$ . more 108; is  $220\frac{1}{2}$ . &c.  
0 more  $31\frac{1}{2}$ ; is  $31\frac{1}{2}$ . more  $112\frac{1}{2}$ ; is 144. more  $220\frac{1}{2}$ . is 364. more  $355\frac{1}{2}$ ; is 720.

Here remember that the Transverse diameter is found, by the 9 of the 23 Proposition. of *Stereom. Prop.* Also the parameter found by the converse of the first part of the 11 Prop. of *Stereom. Prop.* The parameter of the parabolike conoid is found, by the converse of the 9 Prop. Of *Stereom. Prop.*

## XVIII. Note.

### Cautions Concerning Reduction.

I

If it be to calculate pyramids whether Regular or Irregular, whole or frustums; the third, second and first differences are to be found as above: then Reduce those differences into Gallons and parts of a Gallon, or Barrells, or parts of a barrells;

Thus

Suppose 288 cubick inches make one Gallon, and 36 Gallons make one Barrell.

Then,

If the measure be taken in inches, divide the third, second and first differences by 288, and so there will be three quotients in Gallons or parts of

of a Gallon, then with those three quotients proceede to make the table of solid Segments, and that table will be in Gallons or parts of a Gallon. If it be to calculate a table in Barrells multiply 288 by 36 and the product will be 10368 the number of cubick inches in one Barrell. Then divide the third, second and first differences by 10368, there will be three quotients in Barrells or parts of a Barrell: Then with these three quotients proceede to make the table of solid Segments.

That table being so made will be in Barrells or parts of a Barrell.

2,

To calculate Cones and Elliptick solids, whether the whole or their frustums.

Haveing found their third second and first differences, as above, and it be to calculate them in cubick inches, Let it be made as 14, is to 11; so is the third difference, to a fourth,

And,

As 14, is to 11; so is the second difference, to a fourth,

Further,

As 14, is to 11; so is the first difference, to a fourth with these three number thus found, proceede to make the table of solid Segments, and that table will be in cubick inches.

To calculate these solids in Gallons.

Multiply 14 by 288 the product will be 4032.

Then,

As 4032, to 11; so is the third difference, to a fourth.

And

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And,

As 4032, to 11; so is the second difference, to a fourth.

Further,

As 4032, to 11; so is the first difference, to a fourth.

With these three numbers thus found, proceed to make the table of solid Segments. So that table will be in Gallons.

To calculate these solids in Barrells.

Suppose 288 cubick inches makes one Gallon, and 36 gallons makes one Barrell, then multiply 288, 36 and 14 one into another and they make 145152.

Then,

As 145152, is to 11; so is the third difference, to a fourth.

And,

As 145152, is to 11; so is the second difference, to a fourth.

Further,

As 145152, is to 11; so is the first difference, to a fourth.

With these three numbers thus found make the table of solid Segments: that table will be in Barrells.

### III.

Having found the third, second and first differences of all the squares of one fourth of a sphere, spheroid and hyperbolick Conoid, as in the 12 and 13 notes and the second and first differences of all the squares of one fourth of a parabolick conoid as in the 11 note: they may be Reduced to Circular differences.

Thus,

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Thus,

As 14, is to 11 ; so is the third difference, to a fourth.

And,

As 14, is to 11, so is the second difference, to a fourth.

Further,

As 14, is to 11 ; so is the first difference, to a fourth.

With these numbers thus found make a table, of solid Segments of cubical inches of one fourth of any of these solids.

These solid Segments ought to be multiplied by four, to reduce them to solid Segments of a whole sphere, spheroid, hyperbolick and parabolick conoids; but to shun that work divide 14, by four, and then find the new differences; but because 14 cannot be just divided by four, therefore divide 14, by two, and multiply 11, by two, and then work; Thus,

Then,

As 7, to 22 ; so is that third difference, to a fourth.

And,

As 7 to 22 ; so is that second difference, to a fourth.

Further,

As 7, to 22 ; so is that first difference, to a fourth.

With these numbers thus found, proceed to make tables as is taught in those Notes: tables so made, will be tables of solid Segments of those solids, in cubick inches.

To calculate these solids in Gallons.

Multiply

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Multiply 288 by 14, whose product is 4032;  
one fourth thereof is 1008;

Then,

As 1008, is to 11; so is the third difference,  
to a fourth.

And

As 1008, is to 11; so is the second difference,  
to a fourth.

Further,

As 1008, is to 11; so is the first difference,  
to a fourth.

Tables being made, with numbers thus found;  
according to the former directions in the sphere,  
spheroid, hyperbolick and parabolick conoids,  
will be tables of solid Segments of a whole sphere,  
spheroid, hyperbolick and parabolick conoid, in  
Gallons or parts thereof.

To calculate these solids in Barrells.

Multiply 4032 by 36, the product will be  
145152, one fourth thereof will be 36288;

Then,

As 36288, is to 11; so is the third difference,  
to a fourth.

And,

As 36288, is to 11; so is the second difference,  
to a fourth.

Further,

As 36288, is to 11; so is the first difference,  
to a fourth.

Tables being made, with numbers thus found,  
according to the former directions, will be tables  
of solid Segments in Barrells. &c.

Then,



Then,

Using a Rod or Ruler equally divided into inches as in scholium the first, the number of Gallons or Barrells may speedily be obtained.

As for the just magnitude of the Gallon, it is not my businesse to dispute; that being determined by custom or Authority: I took 288 onely for Example sake.

## XIX. Note.

*In a Rank of numbers having equal differences.*

Let the first term in the Rank be  $Z$ , its square  $ZZ$ . the second term  $2Z$ , its square  $4ZZ$ , therefore the first of the first differences is  $3ZZ$ , the third term in that Rank  $3Z$ , its square  $9ZZ$ , then  $9ZZ$ , Less  $4ZZ$ , the second of the first differences  $5ZZ$ , therefore  $5ZZ$  Less  $3ZZ$  the second difference will be  $2ZZ$ .

Further,

The fourth term in that Rank is  $4Z$ , its square is  $16ZZ$ , then  $16ZZ$  Less  $9ZZ$  the third of the first differences  $7ZZ$ ; again,  $7ZZ$  Less  $5ZZ$  the second difference is  $2ZZ$ .

Hence it follows,

That the second difference is equall to the square of the first term doubled.

Or also,

The second difference is equall to the square

C

et

of the difference of two of the terms, (in order taken) doubled.

By the same method we find that the third differences in a Rank of cubes are equal, and the third difference is equal to the first term multiplied by 6.

Or,

The third difference is equal to the cube of the difference of two of the terms, taken in order, multiplied by 6.

The index and equal difference, of every power agrees; to wit, the index of the square is 2, and the second differences are equal. The index of the cube is 3, and the third differences are equal. The index of the square squared is 4, and the fourth differences are equal. &c.

The equal difference of every power, is complicated from the index of that power, and the equal difference of the next Lesser power.

Let the Rank be in naturall order, Thus, 1, 2, 3, 4. &c.

The indices of the powers, Thus.

|   |    |     |      |       |
|---|----|-----|------|-------|
| 1 | 2  | 3   | 4    | 5     |
| Z | ZZ | ZZZ | ZZZZ | ZZZZZ |

A unity the equal difference in that naturall Rank, whose square is 1, which multiplied by 2 the index of the square the product is 2, the equal difference in the squares. 3, the index of the cube multiplied by 2 the equal difference in the squares, the

the product is 6, the equal difference in the cubes.  
 4 the index of the square squared multiplied by  
 6 the product is 24 the equal difference in the  
 square squared, &c.

If the Rank be in order thus, 2, 4, 6, 8, &c.

2 the equal difference of this Rank whose  
 square is 4; multiplied by 2 the index of the square  
 the product is 8; the equal difference of the  
 squares in such a Rank. Because the equal difference  
 of the Rank is 2, therefore the indices are  
 to be doubled, &c.

And the equal differences of the powers in  
 such a Rank will be 8, 48, 384, &c.

## XX. Note.

*For the more easier calculation of the second sections  
 of the sphere and spheroid; worke, Thus.*

From the double of the superficies of the  
 triangle BZN, subtract the superficies of the  
 triangles BZGD and NZPA, the Remainder  
 will be the superficies of the triangles BZPA  
 and NZGD, the areas of these two triangles  
 being subtracted from the area of the triangle  
 NZB, the Remainder will be the superfice of the  
 triangle ZGDAP.

F I N I S.

By *John Baker*, living in *Barmonsej-street* in *Southwark*, over against the *Princes-Armes*, is Taught *Arithmetick*, both in whole numbers and fractions, *Decimal*, *Logarithmetical* and *Algebraical*, *Geometry*, *Trigonometry*, *Astronomy*, the use of *Globes*, *Navigation*, *Measuring*, *Gageing*, *Dyalling*, &c. Also the *Construction* and use of all the usual lines put upon *Rules* or *Scales*, He also teacheth how to find the (*Length* and) *Spreading* of a *Hip-rafter*, only by a *Line of Chords* of singular use for *Carpenters*, a way not as yet vulgarly known amongst *Workmen*.

## Faults Escaped in the Impression of *Stereometrical Propositions.*

Page 1, line 18, for and Z Read and H. p. 10, l. 23, for 56, r. 58. p. 34, l. 25, after RI put. p. 43, for 297232, r. 297432. p. 45, for 18, r.  $\sqrt{432}$ . p. 46, l. 1, for XVI, r. XVII. and l. 21, for AE, 16; r. AF, 6; p. 48, l. 6, for 634. r. 624. p. 51, l. 22, for diameter, r. semidiameter. p. 58, l. 25, for ZB, r. XB; p. 63, l. 1, for XIII. r. XXIII. p. 100, l. 17, for parameter, r. diameter. p. 102 and 103 for as 4 to 3, r. as 3 to 2. p. 105, l. vlt. for  $Z+2$ . r.  $Z-2$ . p. 106, l. 4, for  $Z=$ , r.  $Z-\frac{3}{5}$ . and l. 25, for 89, r. 98.

p. 7 against 12 in the first col. in the sec. r. 576. and in the fifth colum f. 96 r. 99. p. 13. in the second col. f. 46. r. 36. and in the same col. f. 2216, r. 2916.



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